ANALYSIS OF PRICE INDICES OF ELECTRICAL APPLIANCES IN SOUTH AFRICA

by

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DECLARATION

I declare that the dissertation hereby submitted to the University of Limpopo, for the degree of Master of Science in Statistics has not previously been submitted by me for a degree at this or any other university; that it is my work in design and in execution, and that all material contained herein has been duly acknowledged.

Maluleke, H (Mr) **Date**

DEDICATION

This project is dedicated to my mother, Maria.

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ABSTRACT

An analysis of price indices of electrical appliances in South Africa is performed using monthly data from Statistics South Africa for the period January 1998 to December 2010, with 2005 as a base year. Time series analysis (exponential smoothing and ARIMA) and neural networks are employed in developing forecasting models. The results for single, double and triple exponential smoothing are compared and triple exponential smoothing is found to be the best model amongst the three to forecast the electrical price indices in South Africa. ARCH models were also employed for the variable that failed to pass the requirements from ARIMA. Comparing neural networks, ARIMA and triple exponential smoothing results, neural networks is found to be the best model for forecasting price indices of electrical appliances. Regression analysis was then applied to the lighting equipment variable to check for a monthly effect after its plot depicted some seasonality pattern. Only the month of February did not have an impact or an effect on time since it was found not to be significantly different from zero. Multivariate time series is also applied in checking the correlation between the variables.

Keywords: Time series analysis, ARIMA, ARCH, multiple linear regression, exponential smoothing, neural networks, electrical price indices.

LIST OF ABBREVIATIONS

ACF	Autocorrelation function
AIC	Akaike information criterion
AICC	Akaike information corrected criterion
ANOVA	Analysis of variance
AR	Autoregressive
ARCH	Autoregressive heteroskedasticity
ARIMA	Autoregressive integrated moving average
ARMA	Autoregressive moving average
DES	Double exponential smoothing
DOE	Department of Energy
EER	Energy efficiency ratio
ES	Exponential smoothing
FPEC	Final prediction error criterion
GARCH	Generalised autoregressive conditional heteroskedasticity
GDP	Gross domestic product
HQC	Hannan-Quinn criterion
IMF	International Monetary Fund
ISIC	International SIC
MA	Moving average
MAE	Mean absolute error
MAPE	Mean absolute percentage error
ME	Mean error
MINIC	Minimum information criterion
ML	Maximum likelihood
MPE	Mean percentage error
PACF	Partial autocorrelation function

PPI	Production Price Index
REIPERA	Research on Electricity in the Integrated Provision of Energy to Rural Areas
RMSE	Root mean square error
SBC	Schwarz Bayesian criterion
SEER	Seasonal energy efficiency ratio
SES	Single exponential smoothing
SIC	Standard Industrial Classification
SMA	Seasonal moving average
Stats SA	Statistics South Africa
TES	Triple exponential smoothing
TSD	Technical support document
VAR	Vector of autoregressive
VARMA	Vector of autoregressive moving average
VMA	Vector of moving average

CHAPTER 1: INTRODUCTION

1.1 Title

Analysis of price indices of electrical appliances in South Africa.

1.2 Introduction

Each year Statistics South Africa (Stats SA) publishes nearly 300 statistical releases. Most of the data collected by Stats SA is hardly ever used for research purposes. One of these statistical releases is “Manufacturing: Production and Sales”, for which data has been collected monthly since 2005 from the Business Register developed from the units registered for value added tax and income tax obtained from the South African Revenue Service (Statistics South Africa, 2005-2012).

The physical volume of manufacturing production is calculated using the results of the monthly survey on “Manufacturing: Production and Sales”. The indices produced from this survey play an important role since they are used as an indicator of the real level of manufacturing activity in the economy. They are used in assessing the state of the economy and in formulating economic policy. They are also important inputs in estimation of the gross domestic product (GDP). The index of physical volume of manufacturing production, also known as the production index, is a statistical measure of the change in the volume of production (Statistics South Africa, 2005-2012).

The data from the “Manufacturing: Production and Sales” survey adhere to the Special Data Dissemination Standard of the International Monetary Fund (IMF). The statistical unit is the enterprise which is classified according to the Standard Industrial Classification (SIC) of all economic activities, which is based on International SIC (ISIC) with suitable adaptations for local conditions (Statistics South Africa, 2005-2012).

For our study, we analyse price indices of electrical appliances in South Africa using Stats SA’s “Manufacturing: Production and Sales” data. The calculation of the monthly production indices is based on the value of sales of products and articles manufactured, and change in monthly value of stocks of manufactured products. This is after the effect of price changes

has been eliminated through deflation using appropriate indices of the Production Price Index (PPI) (Statistics South Africa, 2000-2012).

With Eskom, the main supplier of electricity in South Africa, on a massive plan for a rapid expansion of electricity generation, energy saving has become an important component of South Africa's energy policy. South Africa's electricity prices have increased enormously in recent years due to supply shortages. With a larger uncertainty and a more rapid change in today's price indices of electrical appliances, a heavier role to play lies within predicting future prices (Karnani, 2007). Forecasts enable one to anticipate the future and plan accordingly. Good forecasts are the basis for short-, medium- and long-term planning with respect to consumers and government regarding price indices of electrical appliances in South Africa. The challenges surrounding power crisis such as blackouts, power failures, load shedding etc., have increased the necessity to explore more energy savings and less costly electrical appliances.

1.3 Research problem

Prices of electrical appliances keep on increasing depending on the demand, and this makes life difficult as they may not be affordable, particularly by most people in developing countries. This study investigates the effect of prices of electrical appliances over time in South Africa and the impact of price increases on disposable income of consumers and also predict the future prices of electrical appliances.

1.4 Purpose of the study

1.4.1 Aim

The aim of the study is to analyse the price indices of electrical appliances in South Africa.

1.4.2 Objectives

The objectives of this study are to:

a. determine the monthly effect on the price of electrical appliances in South Africa.
b. determine the impact of price increases on disposable income of consumers.
c. compare time series techniques in predicting the best model for forecasting.
d. predict the future price indices for electrical appliances by building forecasting models.

1.5 Significance of the study

Today electrical appliances are used throughout our homes, at work, in communication, in transformation, and in medicine and science. Therefore, electrical appliances play an important role in our lives. People's lives depend more on electrical appliances, as it is not easy to live without these devices. The problem arises when we are failing to afford electrical appliances due to their high prices. This study is intended to establish possible causes of the changes in prices of electrical appliances for both companies and households.

With electricity price hikes in recent years, and the majority of households in South Africa being poor, it becomes necessary to research on cost-effective energy saving electrical appliances, thus rendering our study relevant.

CHAPTER 2: LITERATURE REVIEW

2.0 Literature review

This chapter gives a brief summary of the work done by other researchers on price indices of electrical appliances.

2.1 Prices of Electrical Appliances in United States and Japan

Between 1955 and 1994, Nakagami (1996) conducted a study on lifestyle change and energy use in Japan with respect to household equipment such as air conditioner and colour TV. The results of the study revealed that in 1970, 26.3% of Japanese households had a colour TV, 5.9% had an air conditioner, and 22.1% had a passenger car. In 1970, the proportions of households owning these two pieces of equipment (i.e. colour TV and air conditioner) were 79.1% and 37.4%, respectively. The combined penetration ratio of kerosene stoves and gas stoves was 122.5%, indicating that each household had approximately one space heater.

Nomura and Jorgenson (2005) conducted a study on industry origins of Japanese economic growth. The purpose of their study was to quantify impact of IT production on Japanese economy. They compared IT prices in the US and Japan at the SIC (Standard Industrial Classification) three-digit, four-digit, and five-digit levels. Comparing the US and Japanese price data for Personal Computers, General Purpose Computers and Servers at the five-digit level from 1995 to 2003, the gap between the two countries was not large if the index numbers are constructed by aggregation over the most detailed items available. By adjusting the index number formula and aggregation weights for the Wholesale Price Index or Cooperate Goods Price Index to be consistent with the Bureau of Economic Analysis output price, the resulting price declines for Electronic Computers were comparable. During 1995-2003 prices decreased by 29.3% per year in the US, compared to 27.0% per year in Japan. At the three-digit level the price of Electronic Computers and Peripheral Equipment decreased by 23.8% per year in the US compared to 15.0% per year in Japan. A significant portion of the price gap at the three-digit level could be explained by the Peripheral Equipment price, which falls less rapidly in Japan and had a larger share of total output when exports were included. The researchers concluded that computer prices at the SIC three-digit, four-digit, and five-digit levels in the US and Japan were appropriately adjusted for quality change after 1995.

According to American Council for an Energy-Efficiency Economy, in 1995, American homes used almost 25% of the energy consumed in the United States (US) (Amann *et al*., 2007). About 80% of that energy was used in single-family homes, 15% in multi-family homes (such as apartments), and 5% in mobile homes. Their study also emphasised that although residential energy use had steadily increased over the past 25 years, it had increased at a slower rate than the rate of population increase. However, many efficiency gains were being offset by increases in the number of electronics and appliances in the average home. Residential air conditioning accounted for around 5% of the electricity consumed in the US Residential air conditioning technologies which included window air conditioners, central air conditioners, heat pumps, passive cooling, and alternatives to air conditioning (including fans). Air conditioner efficiency is rated using the SEER (seasonal energy efficiency ratio) and EER (energy efficiency ratio) metrics. The higher these numbers are, the more efficient the air conditioner.

Between 1980 and 2000, Dale *et al*. (2009), conducted a study to provide a retrospective evaluation to assess the validity of Department of Energy (DOE) estimates of the consumer cost of efficiency standards against actual price data. The DOE in the US announced and implemented minimum efficiency standards for a variety of residential appliances, including room air conditioners, central air conditioners, refrigerators and clothes washers. Accompanying each announcement, DOE issued a technical support document (TSD) for the rule making. As part of these studies, DOE contractors forecasted the retail price increases that would result in a market of more energy efficient, and presumably, more costly equipment. This estimate was generally performed using an engineering approach, that is, by assessing the material and labour costs to manufacturers associated with implementing energy efficiency technology. Inflation-adjusted prices were assumed constant over time, and uniform retail markups were applied. The TSD price estimates are integral in assessing the cost impact to the consumer, the payback period and the national impacts of higher appliance standards. Life cycle cost and payback period are calculated based on estimates of incremental costs related to efficiency improvement. Standards tend to be set at the highest cost-effective level.

In 1982 TSD predicted that, in the US, a unit efficiency increase of small, medium and large appliances would raise prices by $55, $132 and $272 dollars, respectively (Dale *et al*., 2009).

The study was conducted using regression analysis that revealed the market price of efficiency to range from one-third to two-thirds of the TSD forecast price. A one-unit increase in the EER of a small, medium and large room air appliance was estimated to increase price by an average of $32, $58 and $66 dollars, respectively. Again the 1990 TSD predicted the price of efficiency to be $56 for small appliances, $66 for medium appliances and $37 for large appliances. The regression analysis suggested that the market price of efficiency fell to $9 for small appliances and to $41 for medium sized appliances, and that the 1987-1993 price of efficiency was $52 higher than the 1990 TSD prediction (Dale *et al*., 2009).

2.2 Prices of Electrical Appliances in China

The Konka Group, one of China's major home appliance makers, reported 62% growth year on year in overseas colour TV sales for January to June 2006 (Xinhua News Agency, 2006). Without elaborating on the company's overseas sales income, Konka had also sold more than one million cellular phones overseas during the period. According to the report, in the first half of 2005, Konka's income from overseas colour TV sales reached 610 million yuan (US$76.25 million), with a gross profit margin of about 9%. In 2005, Konka's sales income from overseas colour TV sales totalled 1.479 billion yuan and made up 13% of the company's total revenue (Xinhua News Agency, 2006).

Kemmler (2007) presented a paper that examined the factors that influence household electrification in India. The 2002 Johannesburg Summit of the United Nations (UN) stated that "To implement the goal accepted by the international community to halve the proportion of people living on less than US$1 per day by 2015, access to affordable energy services is a prerequisite", Kemmler (2007:15). In particular, electricity with its wide range of applications may be important for development. Various studies have attempted to measure the social and economic benefits of electrification for rural populations. These studies highlight lower costs and higher use of household appliances (lighting, radio, TV), improved returns on education, wage income and home business productivity, time savings for household chores and ability to use electric pump sets as main benefits of electricity access. The increased recognition of the benefits of rural electrification for poverty alleviation and development led to a new emphasis on ensuring that rural households have access to, and adopt electricity.

Cai and Jiang (2008) identified five sites in China, where they studied the energy consumption spectra of household change. The five sites ranged from remote mountains to town areas.

The main purpose of their study was to determine the differences in energy consumption between rural and urban households and to assess its conservation implications. The study revealed that the amount of electricity used for entertainment and electrical appliances was bigger in urban area, whereas the quantity used for cooking was higher in rural areas. In the Laoxiancheng village, almost all energy is spent on cooking and heating, no energy was used for recreation. In Houzhenzi Xiang township, a greater part of the energies was used for cooking. A large ratio of energy was used for heating in both Laoxiancheng Village and Houzhenzi Xiang township, following the old traditional custom of burning woods in the winter. Electricity in the Houzhenzi Xiang township was only used for illumination and for some electric appliances like TV. There were almost no customs of heating in the Mazhao town and the Zhouzhi County town in winter. Most of the energy the residents consumed was for cooking. The amount of energy used for lighting and recreation was low. Electricity consumption was a much more important component of energy consumption. Heating in winter and using private vehicles are more popular in the city. With the development of urbanisation and economics, people use less energy for basic necessities of life, such as cooking, and more energy for recreation (Cai and Jiang, 2008).

2.3 Prices of Electrical Appliances in the Philippines

Bensel and Remedio (1995), surveyed residential energy use patterns in Cebu city, Philippines. The purpose of the survey was to quantify household consumption of electricity and other fuels and to determine the major environmental factors that drive fuel-choice and fuel-use patterns in residential sector. The scholars found out that electricity accounts for 20.7% of delivered energy and 50.5% of useful energy consumption in the residential sector of Cebu City. Electricity is used primarily for lighting and the a few major appliances, notably refrigerators, colour TVs, washing machines, air conditioners, and electric fans. During the time of their study residential sector electricity demand in Cebu city was growing faster than the national average, and was still quite far from reaching a saturation point. Electric power shortages have been a hindrance to economic expansion in the province, prompting the Philippine government to undertake projects to interconnect Cebu city's electric power grid with geothermal power plants on the islands of Leyte and Negros.

2.4 Prices of Electrical Appliances in Africa

Wentzel *et al.* (1997) and James and Ntutela (1997) conducted a post-electrification study at Mafefe (Limpopo Province) and Tambo (Eastern Cape Province) to study the use of grid electricity by rural households in South Africa. They established that in these two areas candles and paraffin were still used substantially for lighting even after six months electrification. The main reason for this was that electric lighting was not available in all the rooms in the houses. Eskom generally provides households with ready boards comprising a light and a row of sockets, which allows households immediate access to electricity without the expense of formal house wiring, while reducing the reticulation cost to the utility. A survey conducted at Tambo found that about 76% of households had electric lighting in one room only, while about 14% had lights in two rooms, and only about 10% had lights in more than two rooms (James and Ntutela, 1997). Immediately after electrification the most common fuels for lighting were a combination of electricity and other fuels, while almost 40% of households were using no electric lighting at all. After two years the situation had changed completely, with 79% of households relying solely on electricity, and 21% using it in conjunction with other fuels.

According to Thom (2000), in South Africa (Eastern Cape Province), electric lighting is commonly used by electrified households. Research on Electricity in the Integrated Provision of Energy to Rural Areas (REIPERA) project indicated that a significant percentage of electrified households continued to use other fuels particularly candles, and to a lesser extent, paraffin lamps, for lighting purposes.

Bucini *et al*. (2010) conducted a study to analyse the geographical differences in unit expenditures for domestic energy and to find evidence of an inverted energy ladder with prices of useful energy units. They analysed the energy consumption patterns in Mozambique from a sample of 8377 energy-consuming households surveyed during 2002/2003. The results of their study indicated that urban high-income households were the major consumers of electricity, while poor rural households relied mostly on firewood alone. In other words, the energy ladder concept, associating high incomes with high-grade sources, (biomass) was applicable. The results also indicated that urban households had a higher electricity consumption rate (0.99%) than rural household (0.08%). So, income levels were not the restraining factor in the adoption of electricity as a domestic source.

2.5 Literature reviews about some of the techniques used in the study

2.5.1 Time Series

A time series is an ordered sequence of observations. Although the ordering is usually through time, particularly in terms of some equally spaced time intervals, the ordering may also be taken through other dimensions, such as space. According to Cryer and Clan (2008), time series is used in a variety of fields. In agriculture, annual crop production and prices are observed. In business and economics, daily closing stock prices, weekly interest rates, monthly price indices, quarterly sales and yearly earnings are observed. In engineering, sound, electric signals and voltage are observed. In meteorology hourly wind speeds, daily temperature, and annual rainfall are observed. In the social sciences, annual birth rates, mortality rates, accidents rates and various crime rates are studied.

2.5.2 Neural Networks

According to Kaastra and Boyd (1996), neural networks are universal function approximators that can map any nonlinear function. As such flexible function approximators, they are powerful methods for pattern recognition, classification and forecasting. Neural networks are less sensitive to error term assumptions and can tolerate noise and chaotic components. Other advantages include greater fault tolerance; robustness and adaptability compared to expert systems due to the large number of interconnected processing elements that can be trained to learn the pattern.

According to Thielbar and Dickey (2011), a comparison of linear methods, smooth transition autoregressive methods and autoregressive neural networks performed in Terasvirta (2005) shed some light on neural network estimation problems and estimated forecasts. Furthermore, the claim by some researchers that autoregressive neural networks could estimate trend and seasonality was disputed by Zhang and Qi (2005) in their empirical study of simulated series with seasonality, which showed that neural networks performed much better after the series was adjusted.

2.5.3 The Bootstrap technique

The bootstrap technique can be used to obtain interval forecasts for an autoregressive time series. Bootstrap is a useful technique for three reasons; namely: it is distribution-free; takes into account that the parameters and order of the model are unknown; and improved computer

technology makes the difficult calculations involved with the bootstrap techniques more easier. du Plessis (2000); said that Boraine (2000) showed that the bootstrap results for linear models can be extracted to non-linear time series models.

CHAPTER 3: METHODOLOGY

3.0 Research methodology

This research will be conducted using secondary data of electrical appliances from Statistics South Africa. Statistical techniques such as time series, multivariate time series analysis and regression analysis will be used.

3.1 Univariate time series

Univariate time series is a time series that consists of single measurements recorded over different time intervals. Usually the measurements are recorded at equal spaced time intervals, resulting in a discrete series. The series become a stochastic process if measured along a continuous time interval. The objective of a time series analysis is to determine the relationship between a specific value for a time series and its past values.

3.1.1 Fundamental components

3.1.1.1 The autocovariance and autocorrelation functions

For stationary process $\{Z_t\}$, let the mean $\mathrm{E}(Z_t) = \mu$ and the variance $\mathrm{var}(Z_t) = \mathrm{E}(z_t - \mu)^2 = \sigma^2$, which are constant, and the covariance be $\mathrm{cov}(Z_t, Z_s)$, which are functions only of the time difference absolute values of $t - s$.

Hence in this case, the covariance between Z_t and Z_{t+k} can be written as

$$\gamma_k = \mathrm{cov}(Z_t, Z_{t+k}) = \mathrm{E}(Z_t - \mu)(Z_{t+k} - \mu) \quad \text{.....(3.1)}$$

and the correlation between Z_t and Z_{t+k} as

$$\rho_k = \frac{\mathrm{cov}(Z_t, Z_{t+k})}{\sqrt{\mathrm{var}(Z_t)\mathrm{var}(Z_{t+K})}} = \frac{\gamma_k}{\gamma_0} \quad \text{.....(3.2)}$$

where $\mathrm{var}(Z_t) = \mathrm{var}(Z_{t+k}) = \gamma_o$ is a function of k. γ_k is called the autocovariance function and ρ_k is called the autocorrelation function (ACF) in time series analysis since they represent the covariance and correlation between Z_t and Z_{t+k} from the same process separated only by k time lags.

It is easy to see that for a stationary process the autocovariance function γ_k and the ACF ρ_k have the following properties:

$\gamma_o = \mathrm{var}(Z_t)$; $\rho_o = 1$

The absolute value of $\gamma_k \leq \gamma_o$; the absolute value of $\rho_k \leq 1$

$\gamma_k = \gamma_{-k}$ and $\rho_k = \rho_{-k}$ for all k, i.e. $\gamma_k = \rho_k$

which are even functions and symmetric about the time origin, $k = 0$. This follows from the fact that the time difference between (Z_t and Z_{t+k}) and (Z_t and Z_{t-k}) are the same. Therefore the ACF is often plotted only for the nonnegative lags.

Another important property of the autovariance γ_k and the ACF ρ_k is that they are positive semi definite in the sense that $\sum\sum \alpha_i\alpha_j\, \gamma_{|t_i - t_j|} \geq 0$ and $\sum\sum \alpha_i\alpha_j\, \rho_{|t_i - t_j|} \geq 0$ for any set of time points $t_1, t_2, \dots \dots .., t_n$ and any real numbers $\alpha_1, \alpha_2, \dots .., \alpha_n$.

3.1.1.2 The partial autocorrelation function

In addition to the autocorrelation between Z_t and Z_{t+k} it may be needed to investigate the correlation between Z_t and Z_{t+k} after mutual linear dependency on the intervening variables $Z_{t+1}, Z_{t+2}, \dots \text{and } Z_{t+k-1}$ has been removed. This gives rise to the following conditional correlation

$$\mathrm{corr}(Z_t, Z_{t+K}/Z_{t+1}, \dots ., Z_{t+k-1}) \quad \text{.....(3.3)}$$

and is usually referred to as the partial autocorrelation in time series analysis.

3.1.1.3 White noise

Wei (1990) defines a process $\{a_t\}$ as white noise process if it is a sequence of uncorrelated random variables from a fixed distribution with constant mean $E\{a_t\} = \mu_a$ usually assumed to be zero, constant variance $\mathrm{var}(a_t) = \sigma^2$ and $\gamma_k = \mathrm{cov}(a_t, a_{t+k}) = 0$ for all $k \neq 0$. By definition, it immediately follows that a white noise process $\{a_t\}$ is stationary with the autocovariance function.

$$\gamma_k = \begin{cases} \sigma_a^2 & k = 0 \\ 0 & k \neq 0 \end{cases} \quad \text{.....(3.4)}$$

and the ACF

$$\rho_k = \begin{cases} 1 & k = 0 \\ 0 & k \neq 0 \end{cases} \quad \text{.....(3.5)}$$

and partial autocorrelation function (PACF)

$$\phi_{kk} = \begin{cases} 1 & k = 0 \\ 0 & k \neq 0 \end{cases} \quad \text{.....(3.6)}$$

When talking about autocorrelation and partial autocorrelations, we are only referring to ρ_k and ϕ_{kk}, for $k \neq 0$. The basis phenomenon of the white noise process is that its ACF and PACF are identically equal to zero.

3.1.2 Stationary time series models

In time series analysis, there are two useful representations to express a time process. One is to write a process Z_t as a linear combination of a sequence of uncorrelated random variables, i.e.,

$$Z_t = \mu + a_t + \psi_1 a_{t-1} + \psi_2 a_{t-2} + \cdots = \mu + \sum_{j=0}^{\infty} \psi_j a_{t-j} \quad \text{..... (3.7)}$$

where $\psi_0 = 1$, $\{a_t\}$ is a zero mean white noise process, and $\sum \psi_j^2 < \infty$. Here and in the following an infinite sum of random variables is defined as a limit in quadratic mean (mean square) of the finite partial sums. Thus, Z_t in equation (3.7) is defined such that

$$E\left[\left(\dot{Z}_t - \sum_{j=0}^{\infty} \psi_j a_{t-j}\right)^2\right] \to 0 \text{ as } n \to \infty,$$

where $\dot{Z}_t = Z_t - \mu$. By introducing the backshift $B_t x_t = x_{t-j,}$ equation (3.7) can be written in the compact form

$$\dot{Z}_t = \psi(B) a_t \quad \text{.....(3.8)}$$

where $\psi(B) = \sum_{j=0}^{\infty} \psi_j B^j$.

It is easy to show that for the process in equation (3.7)

$$E(Z_t) = \mu, \quad \text{.....(3.9)}$$

$$\text{var}(Z_t) = \sigma_a^2 \sum_{j=0}^{\infty} \psi_j^2, \quad \text{.....(3.10)}$$

and

$$E\left(a_t Z_{t-j}\right) = \begin{cases} \sigma_a^2, & \text{for } j = 0, \\ 0, & \text{for } j > 0. \end{cases} \quad \text{.....(3.11)}$$

Hence,

$$\gamma_k = E\left(\dot{Z}_t \dot{Z}_{t+k} \right) = \sigma_a^2 \sum_{i=0}^{\infty} \psi_i \psi_{i+k} \quad \text{.....(3.12)}$$

and

$$\rho_k = \frac{\sigma_a^2 \sum_{i=0}^{\infty} \psi_i \psi_{i+k}}{\sum_{i=0}^{\infty} \psi_i^2} \quad \text{.....(3.13)}$$

The autocovariance and autocorrelation functions in equations (3.12) and (3.13) are functions of the time difference k only. Because they involve infinite sums, to be stationary we have to show that γ_k is finite for each k. Now,

$$|\gamma_k| = \left| E\left(\dot{Z}_t \dot{Z}_{t+k} \right) \right| \le \sqrt{\text{var}(Z_t)\,\text{var}(Z_{t+k})} = \sum_{j=0}^{\infty} \psi_j^2.$$

Hence, $\sum_{j=0}^{\infty} \psi_j^2 < \infty$ is a required condition for the process in equation (3.7) to be stationary. The form in equation (3.7) is called a moving average (MA) representation of a process.

Another useful form is to write a process Z_t in an autoregressive (AR) representation, in which the value of Z is regressed at time t on its own past values plus a random shock, i.e.,

$$\dot{Z}_t = \pi_1 \dot{Z}_{t-1} + \pi_2 \dot{Z}_{t-2} + \cdots + a_t$$

Or, equivalently,

$$\pi(B)\dot{Z}_t = a_t, \quad \text{.....(3.14)}$$

where $\pi(B) = 1 - \sum_{j=1}^{\infty} \pi_j B^j$, and 1+ $\sum_{j=1}^{\infty} |\pi_j| < \infty$.

The autoregressive representation is useful in understanding the mechanism of forecasting. It is not every stationary process that is invertible. For a linear process $Z_t = \psi(B)\dot{a}_t$ to be invertible so that it can be written in terms of the AR representation, the roots of $\psi(B) = 0$ as a function of B must lie outside the unit circle. It should be noted that an invertible process in not necessarily stationary. For the process that for the presented in equation (3.14) to be stationary, the process must be able to be rewritten in a MA representation, i.e.,

$$\dot{Z}_t = \frac{1}{\pi(B)} a_t = \psi(B) a_t, \quad \text{.....(3.15)}$$

Such that the condition $\sum_{j=0}^{\infty} \psi_j^2 < \infty$ is satisfied. To achieve that, the required condition is that the roots of $\pi(B) = 0$ all lie outside the unit circle.

3.1.3 Stationary time series: The Autoregressive process

In the autoregressive representation of a process, if only a finite number of weights π are nonzero, i.e. $\pi_1 = \phi_1, \pi_2 = \phi_2, \ldots\ldots, \pi_p = \phi_p$ and $\pi_k = 0$ for $k > p$, then the resulting process is said to be an autoregressive process (model) for order *p*, which is denoted as AR(*p*).

It is given by:

$$\phi_P(B) = 1 - (\phi_1 B - \cdots - \phi_P B^P). \quad \text{.....(3.16)}$$

The AR processes are useful in describing situations in which the present value of a time series depends on its preceding values plus a random shock.

3.1.3.1 The first order autoregressive AR(1) process

For the first order autoregressive process AR(1), the formulae can be written as $(1 - \phi_1 B)\dot{Z}_t = a_t$. The process is always invertible. To be stationary the root of $(1 - \phi_1 B) = 0$ must be inside the unit circle. That is, for a stationary process, we have $|\phi_1| < 1$. The AR(1) process is sometimes called the Markov process because the value of $\dot{Z}_t$ is completely determined by the knowledge of $\dot{Z}_{t-k}$.

The ACF of the AR(1) process

The autocovariances are obtained as follows:

$$\mathrm{E}(\dot{Z}_{t-k}\dot{Z}_t) = \mathrm{E}(\phi\dot{Z}_{t-k}\dot{Z}_{t-1}) + (\dot{Z}_{t-k}a_t) \quad \text{....................................(3.17)}$$

and the ACF becomes $\rho_k = \rho_1\rho_{k-1} = \phi_1^k,\ k \geq 1$ where $\rho_0 = 1$. When $|\phi_1| < 1$ and the process is stationary, the ACF exponentially decays in one of two forms depending on the sign of ϕ_1. If 0< $\phi_1 < 1$, the autocorrelations are positive. If $-1 < \phi_1 < 0$, the sign of the autocorrelations shows an alternating pattern beginning with a negative value.

The PACF of the AR(1) process

For an AR(1) process, the PACF form is

$$\phi_{kk} = \begin{cases} \rho_1 = \phi_1 & k = 1 \\ 0 & k \geq -2 \end{cases} \text{..(3.18)}$$

The PACF of the AR(1) process shows a positive or negative spike at lag 1 depending on the sign of ϕ_1.

3.1.3.2 The second order autoregressive AR(2) process

The second order autoregressive AR(2) process is given by:

$$(1 - \phi_1 B - \phi_2 B^2)\dot{Z}_t = a_t \quad \text{...(3.19).}$$

The AR(2) process, as a finite autoregressive model, is always invertible. To be stationary, the roots of $\phi(B) = (1 - \phi_1 B - \phi_2 B^2) = 0$ must lie outside of the unit circle.

The ACF of the AR(2) process

The autocovariance is obtained by multiplying Z_{t+k} on both sides of $\dot{Z}_t = \phi_1\dot{Z}_{t-1} + \phi_2\dot{Z}_{t-2}a_t$ and taking the expectations,

$$E(\dot{z}_{t-k}\dot{z}_t) = \phi_1 E(Z_{t-k}z_{t-1}) + \phi_2 E(z_{t-1}z_{t-2}) + E(z_{t-k}a_t),$$

$$\gamma_k = \phi_1\gamma_{k-1} + \phi_2\gamma_{k-2}\,; \quad k \geq 1 \quad \text{..(3.20)}$$

hence the ACF becomes

$$\rho_k = \phi_1\rho_{k-1} + \phi_2\rho_{k-2}; \quad k \geq 1. \quad \text{..(3.21)}$$

3.1.3.3 The general p[th] order autoregressive AR(p) process

The general p^{th} order autoregressive AR(*p*) process is expressed as

$$\left(1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^P\right)\dot{z}_t = a_t \quad \text{.....(3.22)}$$

or

$$\dot{Z}_t = \phi_1 \dot{Z}_t + \phi_2 \dot{Z}_t - \cdots - + \phi_p \dot{Z}_{t-p} + a_t. \quad \text{.....(3.23)}$$

The ACF of the general AR(*p*) process is given by:

$$\dot{Z}_t = \phi_1 \dot{Z}_{t-1} + \phi_2 \dot{Z}_{t-2} - \cdots - + \phi_p \dot{Z}_{t-p} + a_t. \text{.....(3.24)}$$

For the ACF of the general AR(*p*) process, equation (3.24) is multiplied by Z_{t-k} on both sides to obtain:

$$\dot{Z}_t Z_{t-k} = \phi_1 \dot{Z}_{t-1} Z_{t-k} + \phi_2 \dot{Z}_{t-2} Z_{t-k} - \cdots - + \phi_p \dot{Z}_{t-p} Z_{t-k} + Z_{t-k} a_t \text{ .....(3.25)}$$

and taking the expected value gives:

$$\gamma_k = \phi_1 \gamma_{k-1} + \cdots + \phi_p \gamma_{k-p}\,; \quad k \geq 0 \quad \text{..... (3.26)}$$

and the ACF is

$$\rho_k = \phi_1 \gamma_{k-1} + \cdots + \phi_p \gamma_{k-p}\,; \quad k \geq 0 \quad \text{.....(3.27).}$$

<u>The PACF of the general AR(*p*) process</u>

By using the fact that $\rho_k = \phi_1 \gamma_{k-1} + \cdots + \phi_p \gamma_{k-p}$; for $k \geq 0$, the PACF will vanish after lag *p*. This is a useful property in identifying an AR model for the time series model building.

3.1.4 Moving average (MA) process

The MA representation of a process is given by:

$$\psi_1 = -\theta_1, \psi_2 = -\theta_2 \ldots \psi_q = -\theta_q \quad \text{.....(3.28)}$$

and $\psi_k = 0$, if only a finite number of ϕ weights are nonzero.

This MA process is invertible if the roots of $\theta(B) = 0$ lie outside of the unit circle. MA processes are useful in describing phenomenon in which events produce an immediate effect that only lasts for short periods of time.

3.1.4.1 The first order MA(1) process

The first order MA(1) process is given by:

$$\dot{Z}_t = (1 - \theta_1 B)a_t \quad \text{.....(3.29)}$$

where $\{a_t\}$ is a zero mean white noise process with constant variance σ_a^2 and the mean of $\{\dot{Z}_t\}$ is $\mathrm{E}[\dot{Z}_t] = 0$ and hence $\mathrm{E}(Z_t) = \mu$.

The ACF of MA(1) process is given by:

$$\gamma_k = \begin{cases} (1 + \theta_a^2) & k = 0, \\ -\theta, \sigma_a^2 & k = 1, \\ 0 & k > 1. \end{cases} \quad \text{.....(3.30)}$$

and the ACF becomes

$$\rho_k = \begin{cases} \frac{-\theta_1}{1+\theta_1^2}, & k = 1, \\ 0, & k > 1, \end{cases} \quad \text{.....(3.31)}$$

which cuts off lag 1 because $1 + \theta_1^2$ is always bounded. The MA(1) process is always stationary.

PACF of the MA(1) process is given by:

$$\phi_{kk} = \frac{\theta_1^k(1-\theta_1^2)}{1-\theta_1^2(k+1)} \text{ for } k \geq 1. \quad \text{.....(3.32)}$$

3.1.4.2 The second order MA(2) process

The second order MA(2) process is given by:

$$\dot{z}_t = (1 - \theta_1 B - \theta_2 B^2)a_t \quad \text{.....(3.33)}$$

where $\{a_t\}$ is a zero mean white noise process.

<u>The ACF of the MA(2) process</u>

The autocovariances of the MA(2) model are

$$\gamma_0 = (1 + \theta_1^2 + \theta_1^2)\sigma_a^2$$

$$\gamma_1 = -\theta_1(1 - \theta_2)\sigma_a^2$$

$$\gamma_2 = -\theta_2\sigma_a^2$$

and

$$\gamma_k = 0 \qquad k > 0.$$

The ACF is

$$\rho_k = \begin{cases} \frac{-\theta_1(1-\theta_2)}{1+\theta_1^2+\theta_2^2}, & k = 1 \\ \frac{-\theta_2}{1+\theta_1^2+\theta_2^2}, & k = 2 \\ 0, & k > 2 \end{cases} \qquad \text{.....(3.34)}$$

and cuts off at lag 1.

3.1.4.3 The general q[th] order MA(q) process

The general q^{th} order moving average process is

$$\dot{z}_t = (1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_q B^q)a_t \qquad \text{.....(3.35).}$$

The ACF of an MA(q) process cuts off after lag q.

3.1.5 Autoregressive moving average ARMA(*p,q*) process

3.1.5.1 The general mixed ARMA(p,q) process

The general mixed ARMA(p,q) process is one of the finite order moving average and a finite order autoregressive model as it often takes a high order model for good approximation. Thus, in model building it may be necessary to include both autoregressive and moving average terms in the model. This leads to the following useful mixed autoregressive moving average ARMA process:

$$\phi_p(B)\dot{z}_t = \theta_q(B)a_t \qquad \text{.....(3.36)}$$

where

$$\phi_p(B) = 1 - \phi_1 B - \cdots - \phi_p B^p$$

and

$$\phi_q(B) = 1 - \phi_1 B - \cdots - \phi_q B^q$$

For the process to be invertible it is required that the roots of $\theta_q(B) = 0$ lie outside the unit circle, and for the process to be stationary, it is required that the roots of $\phi_p(B) = 0$ lie outside the unit circle. If we assume that $\theta_p(B) = 0$ and $\theta_q(B) = 0$ share no common roots, then the process is referred to as an ARMA(*p,q*) process or model, in which *p* and *q* are used to indicate the order of the associated autoregressive and moving average polynomials, respectively.

The stationary and invertible ARMA process can be written in pure autoregressive representation as follows:

$$\pi(B)\dot{z}_t = a_{t,} \quad \text{.....(3.37)}$$

where

$$\pi(B) = \frac{\phi_p(B)}{\theta_q(B)} = (1 - \pi_1(B) - \pi_2 B^2 - \cdots).$$

This process can also be written as a pure moving average representation as follows:

$$\dot{z}_t = \psi(B)a_t \quad \text{.....(3.38)}$$

where

$$\psi(B) = \frac{\phi_q(B)}{\phi_p(B)} = (1 + \psi_1 B + \psi_2 B^2 + \cdots).$$

ACF of the ARMA(*p, q*) process is given by:

$$\gamma_\kappa = \phi_1\gamma_{k-1} + \cdots + \phi_p\gamma_{k-p} \;\; k \geq (q+1) \quad \text{.....(3.39)}$$

and hence,

$$\rho_\kappa = \phi_1\rho_{k-1} + \cdots + \phi_p\rho_{k-p} \;\; k \geq (q+1) \quad \text{.....(3.40)}$$

Equation (3.31) satisfies the p^{th} order homogeneous difference equation. Therefore, the ACF of an ARMA(*p,q*) model tails off after lag *q* just like an AR(*p*) process, which depends only on the autoregressive parameters in the model. However, first *q* autocorrelations $\rho_q, \rho_{q-1}, \ldots, \rho_1$ depend on both autoregressive and MA parameters in the model and serve as initial values for the parameters.

The PACF of ARMA(*p,q*) process

Because the ARMA process contains the MA process as a special case, its PACF will also be a mixture of exponential decays and/or damped sine waves depending on the roots of $\phi_p(B) = 0$ and $\theta_q(B) = 0$.

3.1.5.2 The ARMA(1,1) process

The ARMA(1,1) process is given by

$$(1 - \phi_1 B)\dot{z}_t = (1 - \theta_1 B)a_t \quad \text{.....(3.41)}$$

or

$$\dot{z}_t = \phi\dot{z}_{t-1} + a_t - \theta_1 a_{t-1}. \quad \text{.....(3.42)}$$

For stationary, $|\phi_1| < 1$ is assumed and for invertibility, $|\theta_1| < 1$ is required. When $\phi_1 = 0$ is reduced to an MA(1) process, and when $\theta_1 = 0$; it is reduced to an AR(1) process, thus, when regarding the AR(1) and MA(1) process as special cases of the ARMA(1,1) process.

3.1.6 Forecasting: Minimum mean square error forecasts

Forecasting is essential for planning and operation control in a variety of areas such as production management, inventory systems, quality control, financial planning, and investment analysis.

Forecasting is one of the important objectives in the analysis of a time series. The term forecasting is used more frequently in recent time series literature than the term prediction.

The general ARIMA(*p,d,q*) model is given by:

$$\phi B(1 - B)^d)z_t = \theta(B)a_t \quad \text{.....(3.43)}$$

where

$$\phi(B) = \left(1 - \phi_1 B - \cdots - \phi_p B^p\right), \theta(B) = \left(1 - \theta_1 B - \cdots - \theta_q B^q\right),$$

and the series a_t is a Gaussian N (0,σ_a^2) white noise process.

3.1.6.1 Minimum mean square error forecasts for ARMA models

To derive the minimum mean error forecasts, first consider the case when *d*=0 i.e., the stationary ARMA model $\phi(B)z_t = \theta(B)a_t$. Because the model is stationary, it can be rewritten in the moving average representations as follows:

$$z_t = \phi(B)a_t \quad = a_t + \psi_1 a_{t-1} + \psi_2 a_{t-2} + \cdots \quad (3.44)$$

where

$$\psi(B) = \sum_{j=0}^{\infty} \psi_j B^j = \frac{\theta(B)}{\phi(B)} \text{ and } \psi_0 = 0.$$

3.1.6.2 Minimum mean square error forecasts for ARIMA models

This is the general non-stationary ARIMA(*p,d,q*) model with $d \neq 0$, i.e.:

$$\phi(B)(1-B)^d z_t = \theta(B)a_t \quad (3.45)$$

where

$$\phi(B) = (1 - \phi_1(B) - \cdots - \phi_p B^p)$$

is a stationary AR operator and

$$\theta(B) = (1 - \theta_1(B) - \cdots - \theta_q B^q)$$

is an invertible MA operator.

3.1.6.3 The ARIMA forecast as a weighted average of previous observations

Smoothing results, such as MA and exponential smoothing are special cases of ARIMA forecasting. ARIMA provides a natural and optimal way to obtain the required weights for forecasting. ARIMA forecasts are minimum mean square error forecasts.

3.1.6.4 Updating forecasts

The updated forecast is obtained by adding to the previous forecast, a constant multiple ψ_1 of the one-step ahead forecast error $a_{n+1} = Z_{n+1} - \hat{z}_n(1)$. This is certainly sensible. For example, when the value Z_{n+1} becomes available and is found to be higher than the previous forecast, resulting in a positive forecast error $a_{n+1} = Z_{n+1} - \hat{z}_n(1)$, the forecast $\hat{z}_n(l+1)$ made earlier by proportionally adding a constant multiple of this error, will naturally be modified.

3.1.7 Model identification

Model identification refers to the methodology in identifying the required transformations such as variance stabilising transformation and differencing transformation.

3.1.7.1 The useful steps for model identification

To illustrate the model identification, consider ARIMA(*p,d,q*) model

$$(1-\phi_1 B - \cdots - \phi_p B^p)(1-B)^d z_t = \theta_0 + (1 - \theta_1 B - \cdots - \theta_q B^q)a_t \text{.....(3.46)}$$

Plot the time series data and choose proper transformations. In any time series analysis, the first step is to plot the data. Through careful examination of the plot we usually get a good idea about whether the series contains a trend, seasonality outliers, non-constant variances and other normal and non-stationarity phenomena. In time series the most commonly used transformations are variance-stabilising transformation and differencing.

Since differencing may create some negative values, then variance stabilising transformation should always be applied before taking differences.

3.1.8 Seasonal process

Time series data may sometimes exhibit strong periodic patterns. This is often referred to as the time series having a seasonal behaviour. This mostly occurs when the data is taken in specific intervals: monthly, weekly and so on. One way to represent such data is through an additive model where the process is assumed to be composed of two parts:

$$y_t = S_t + N_t \quad \text{.....(3.47)}$$

where S_t is the deterministic component with periodicity s and N_t is the stochastic component that may be modeled as an ARMA process, hence y_t can be seen as a process with predictable periodic behaviour with some noise sprinkled on top of it. Since the S_t is deterministic, then

$$S_t = S_{t+s}. \quad \text{.....(3.48)}$$

Applying the $(1-B^s)$ operator to equation (3.38), gives:

$$(1-B^s)y_t = (1-B^s)S_t + (1-B^s)N_t \text{.....(3.49)}$$

$$w_t = (1-B^s)N_t \text{.....(3.50)}$$

where

$(1-B^s)y_t = w_t$ and $(1-B^s)S_t = 0$.

The process w_t can be seen as seasonally stationary. Since an ARMA process can be used to model N_t, in general the model can be written as:

$$\Phi(B)w_t = (1-B^s)\Theta(B)\varepsilon_t \quad \text{.....(3.51)}$$

where ε_t is white noise.

3.1.9 Moving average method

The moving average method is developed based on the assumption that an annual sum of a seasonal time series possesses little seasonal variation. Thus, letting $N_t = P_t + e_t$ be the non-seasonal component of the series, an estimate of the non-seasonal component can be obtained by using a symmetric Moving Average operator. i.e., $\widehat{N}_t = \sum_{i=-m}^{m} \lambda_i Z_{t-i}$ where *m* is a positive integer and the $\lambda_i's$ are constants such that $\lambda_i = \lambda_{-i}$ and $\sum_{i=-m}^{m} \lambda_i = 1$. An estimate of the seasonal component is derived by subtracting $\widehat{N}_t$ from the original series, i.e.,

$\hat{S}_t = Z_t - \widehat{N}_t$. The above estimate may be obtained iteratively by repeating various moving average operators. The series with seasonal fluctuation removed, i.e. $Z_t - \hat{S}_t$, is referred to as the seasonally adjusted series.

3.1.10 ARCH and GARCH models

To check the need of ARCH (autoregressive conditional heteroskedasticity) model, once the ARIMA model is fitted then not only the standard residual analysis and diagnostics checks have to be performed, but also some serial dependence checks for e_t^2 should be made, where $e_t^2 = \zeta_0 + \zeta_1 e_{t-1}^2 + \cdots + \zeta_l e_{t-l}^2 + a_t$, a_t is a white noise sequence with zero mean and constant variance σ_a^2.

To further generalise the ARCH model, let us assume that the error can be represented as

$$e_t = \sqrt{v_t}\,\omega_t \quad \text{.....(3.52)}$$

where ω_t is independent and identically distributed with mean 0 and variance 1, and

$$v_t = \zeta_0 + \zeta_1 e_{t-1}^2 + \zeta_2 e_{t-2}^2 + \cdots + \zeta_l e_{t-1}^2. \quad \text{.....(3.53)}$$

Hence the conditional variance of e_t is

$$\text{var}(e_t \mid e_{t-1}, \cdots) = E(e_t^2 \mid e_{t-1}^2, \cdots) = v_t \quad \text{.....(3.54)}$$

The current conditional variance should also depend on the conditional variance, i.e.

$$v_t = \zeta_0 + \zeta_1 v_{t-1} + \zeta_2 v_{t-2} + \cdots + \zeta_k v_{t-k} + \zeta_1 e_{t-1}^2 + \zeta_2 e_{t-2}^2 + \cdots + \zeta_l e_{t-l} \text{ .....(3.55)}$$

In this notation, the error term e_t is said to follow a generalised autoregressive conditional heteroskedasticity (GARCH) process of order k and l, i.e. GARCH(k, l). In equation (3.46) the model for conditional variance resembles an ARMA.

3.2 Exponential smoothing

Exponential smoothing is a procedure for continually revising a forecast in the light of more recent experience. It assigns exponentially decreasing weights as the observation gets older. There are three types of exponential smoothing methods.

3.2.1 Simple exponential smoothing methods

SES takes the forecast for the previous period and adjusts it using the forecast error, i.e.:

$$F_{t+1} = F_t + \alpha(Y_t - F_t) \text{ .....(3.56)}$$

where

F_{t+1} is the forecast value of period $t+1$

Y_t is the actual value for period t

F_t is the forecast value for period t

α is a (smoothing constant).

The value of the smoothing constant, α has to be between 0 and 1. When α has a value close to 1, it means that the new forecast will indicate a substantial adjustment for the error in the previous forecast and when α is close to zero, the new forecast will indicate very little adjustment. Equation (3.47) can also be written as:

$$F_{t+1} = \alpha Y_t + (1-\alpha)F_t \quad \text{.....(3.57)}$$

where the forecast F_{t+1} is based on weighting the most recent observation (Y_t) with a weight value (α) and weighting the most recent forecast (F_t) with a weight of $(1-\alpha)$.

3.2.2 Holt's linear method (Double exponential smoothing)

According to Makridakis *et al.* (2008), Holt (1957) extended single exponential smoothing to linear exponential smoothing to allow forecasting of the data with the trends. The forecast for Holt's linear exponential smoothing is found using two smoothing constants, α and β, with the values between 0 and 1. The equations of Holt's linear method can be written as:

$$L_t = \alpha Y_t + (1-\alpha)L_{t-1} + b_{t-1} \quad \text{.....(3.58)}$$

$$b_t = \beta(L_t - L_{t-1}) + (1-\beta)b_{t-1} \quad \text{.....(3.59)}$$

$$F_{t+m} = L_t + b_m \quad \text{.....(3.60)}$$

where L_t denotes an estimate of the level of the series at time t and b_t denotes an estimate of the slope of the series at time, t. Equation (3.58) adjusts L_t directly for the trend of the previous period, b_{t-1}, by adding it to the last smoothed value, L_{t-1}. This helps in estimating the lag and brings L_t to the appropriate level of the current data value. Equation (3.59) then updates the trend, which is expressed as the difference between the last two smoothed values. This is appropriate because if there is a trend in the data, new values should be higher or lower than the previous ones. Since there may be some randomness remaining the trend is modified by smoothing with the trend β. Equation (3.60) is used for forecasting ahead. The trend b_t is multiplied by the number of periods ahead to be forecasted m, and added to the base value, L_t.

3.2.3 Holt-Winters trend and Seasonality method (Triple Exponential Smoothing)

According to Makridakis *et al.* (2008), Holt's method was extended again by Winters (1960) to capture seasonality directly. The Holt-Winters method is based on three smoothing equations; one parameter α is for the level, β is for the trend and γ is for seasonality. There are two different Holt-Winters' methods, depending on whether seasonality is modeled in an additive or multiplicative way.

3.2.3.1 Multiplicative seasonality

The basic equations for Holt-Winters for multiplicative method are as follows:

$$L_t = \alpha \frac{Y_t}{S_{t-s}} + (1-\alpha)(L_{t-1} + b_{t-1}) \quad \text{.....(3.61)}$$

$$b_t = \beta(L_t - L_{t-1}) + (1-\beta)b_{t-1} \quad \text{.....(3.62)}$$

$$S_t = \gamma \frac{Y_t}{L_t} + (1-\gamma)S_{t-s} \quad \text{.....(3.63)}$$

$$F_{t+m} = (L_t + b_{tm})S_{t-s} \quad \text{.....(3.64)}$$

where

s is the length of seasonality,

L_t is the level of the series,

b_t denotes the trend,

S_t is the seasonal component,

F_t is the forecast for m periods ahead.

Equation (3.63) is comparable to a seasonal component as a ratio of the current values of the series, Y_t divided by the current single smoothed value for the series, L_t. The data values Y_t do contain seasonality and randomness. In order to smooth this randomness equation (3.61) weights the newly computed seasonal factor with γ and the most recent seasonal number corresponding to the same season with $(1-\gamma)$. Equation (3.62) is the same as Holt's equation (3.59) of Double Exponential smoothing method for smoothing the trend. Equation (3.61) slightly differs from Holts' equation (3.58) in that the first term is divided by the seasonal number S_{t-s}. This is done to deseasonalise Y_t.

3.2.3.2 Additive seasonality

The seasonal component in Holt-Winters' method may also be treated additively. The equations for Holt-Winter's additive method are:

$$L_t = \alpha(Y_t - S_{t-s}) + (1-\alpha)(L_{t-1} + b_{t-1}) \quad \text{.....(3.65)}$$

$$b_t = \beta(L_t - L_{t-1}) + (1-\beta)b_{t-1} \quad \text{.....(3.66)}$$

$$S_t = \gamma(Y_t - L_t) + (1-\gamma)S_{t-s} \quad \text{.....(3.67)}$$

$$F_{t+m} = L_t - b_{tm} + S_{t-s-m} \quad \text{.....(3.68)}$$

Equation (3.66) is identical to equation (3.62) of multiplicative method. The only difference in the other equations from the multiplicative Holt-Winters' method are that the seasonal indices are now added and subtracted instead of taking products and ratios.

3.3 Multivariate time series

Multivariate time series involves several variables that are not only in sequence but also cross-tabulated. As in the univariate case, multivariate or vector ARIMA (autoregressive integrated moving average) models can often be successfully used in forecasting multivariate time series. The first step to be checked is property of stationary.

3.3.1 Multivariate Stationary Process

Suppose that the vector time series $\mathbf{Y}_t = (y_{1t}, y_{2t}, ..., y_{mt})$ consists of m univariate time series. Then $\mathbf{Y_t}$ with finite and second order moments is said to be weakly stationary if

- $E(\mathbf{Y}_t) = E(\mathbf{Y}_{t+s}) = \boldsymbol{\mu}$, constant for all s,
- $\text{Cov}(\mathbf{Y}_t) = E\left[(\mathbf{Y}_t - \boldsymbol{\mu})(\mathbf{Y}_t - \boldsymbol{\mu})'\right] = \boldsymbol{\Gamma}(0)$ and
- $\text{Cov}(\mathbf{Y}_t, \mathbf{Y}_{t+s}) = \boldsymbol{\Gamma}(s)$ depends only on s.

3.3.2 The covariance and correlation matrix function

Let $\mathbf{Z}_t = [Z_{1,t}, Z_{1,t}, \ldots, Z_{m,t}]'$, $t = 0, \pm 1, \pm 2, \ldots$, be an m dimensional joint stationary real-valued vector process so that the mean $E(Z_{i,t}) = \mu_i$ is constant for each $i = 1, 2, \ldots, m$ and the cross-covariance between $z_{i,t}$ and $z_{j,s}$ for all $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, m$ are functions only of the time difference $(s-t)$. The mean vector can be written as:

$$E(\mathbf{Z_t}) = \mu = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_m \end{bmatrix} \quad \text{.....(3.69)}$$

and the covariance matrix

$$\boldsymbol{\Gamma}(k) = \text{Cov}\{\mathbf{Z}_t, \mathbf{Z}_{t+k}\} = E\left[(\mathbf{Z}_t - \mu)(\mathbf{Z}_{t-k} - \mu)'\right]$$

$$= \begin{bmatrix} Z_{1,t-\mu 1} \\ Z_{2,t-\mu_2} \\ \vdots \\ Z_{m,t-\mu_m} \end{bmatrix} \left[Z_{1,t+k-\mu_1}, Z_{2,t+k-\mu_2}, \ldots, Z_{m,t+m-\mu_m} \right]$$

$$= \begin{bmatrix} \gamma_{11}(k) & \gamma_{12}(k) & \cdots & \gamma_{1m}(k) \\ \gamma_{21}(k) & \gamma_{22}(k) & \cdots & \gamma_{2m}(k) \\ \vdots & \vdots & \cdots & \vdots \\ \gamma_{m1}(k) & \gamma_{m2}(k) & \cdots & \gamma_{mm}(k) \end{bmatrix} = \mathrm{Cov}(\mathbf{Z}_{t-k}, \mathbf{Z}_t) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.70)$$

for $k = 0, \pm 1, \pm 2, \ldots,$ $i = 1, 2, \ldots, m,$ and $j = 1, 2, \ldots, m$. As the function of k, $\boldsymbol{\Gamma}(k)$ is called the covariance matrix function for the vector process $\mathbf{Z}_t$.

For $i = j$, $\gamma_{ii}(k)$ is the autocovariance function for i^{th} component process, $Z_{i,t}$.

For $i \neq j$, $\gamma_{ij}(k)$ is the cross-covariance function between $Z_{i,t}$ and $Z_{j,t}$.

3.3.3 Moving average and autoregressive representations of vector process

An *m*-dimensional stationarity vector process $\mathbf{Z}_t$ is said to be a linear process or purely nondeterministic vector of a sequence of m-dimensional white noise random vectors, i.e.:

$$Z_t = \boldsymbol{\mu} + \mathbf{a}_t + \Psi_1 \mathbf{a}_{t-1} + \Psi_2 \mathbf{a}_{t-2} + \ldots = \boldsymbol{\mu} + \sum_{s=0}^{\infty} \Psi_s \mathbf{a}_{t-s} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.71)$$

where $\Psi_0 = \mathbf{I}$ is the $m \times m$ identity matrix, the $\Psi_j{}'s$ are $m \times m$ coefficient matrices, and the $a_t's$ are *m*-dimensional white noise random vectors with zero mean and covariance matrix structure:

$$E[\mathbf{a}_t \mathbf{a}'_{t+k}] = \begin{cases} \sum, & k = 0 \\ 0, & k \neq 0 \end{cases} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.72)$$

where $\sum$ is any arbitrary $m \times m$ symmetric definite matrix.

Thus, although the elements of $\mathbf{a}_t$ at different times are uncorrelated, they may be contemporaneously correlated. Another useful form to express a vector process is through autoregressive representation, in which the value of $\mathbf{Z}$ at time *t* regressed on its own past values plus a vector of random shocks, i.e.,

$$\dot{\mathbf{Z}}_t = \Pi_1 \dot{\mathbf{Z}}_{t-1} + \Pi_2 \dot{\mathbf{Z}}_{t-2} + \cdots + \mathbf{a}_t = \sum_{s=1}^{\infty} \Pi_s \dot{\mathbf{Z}}_{t-s} + \mathbf{a}_t \quad \text{.....(3.73)}$$

where Π_s are $m \times m$ autoregressive coefficient matrices. The vector process is said to be invertible if the autoregressive coefficient matrices are absolutely summable, i.e.; $\sum_{s=0}^{\infty} \left| \Pi_{ij}{}' s \right| < \infty$ for all i and j .

3.3.3.1 The vector autoregressive moving average process

A useful class of parsimonious models is the vector autoregressive moving average ARMA(*p,q*) process, which is given by

$$\dot{\Phi}_p(\mathbf{B})\dot{\mathbf{Z}}_t = \Theta_q(\mathbf{B})\mathbf{a}_t; \quad \text{.....(3.74)}$$

where

$$\Phi_p(\mathbf{B}) = \Phi_0 - \Phi_1\mathbf{B} - \Phi_2\mathbf{B}^2 - \cdots - \Phi_p\mathbf{B}^p$$

and

$$\Theta_q(\mathbf{B}) = \Theta_0 - \Theta_1\mathbf{B} - \Theta_2\mathbf{B}^2 - \cdots - \Theta_q\mathbf{B}^q$$

are the autoregressive and moving average matrix polynomials of order *p* and *q*, respectively, and Φ_0 and Θ_0 are nonsingular $m\,x\,m$ matrices.

3.3.3.1.a Vector AR(1) models

The vector AR(1) model is given by:

$$(1 - \Phi_1\mathbf{B})\dot{\mathbf{Z}}_t = \mathbf{a}_t \quad \text{.....(3.75)}$$

or

$$\dot{\mathbf{Z}}_t = \Phi_1 \dot{\mathbf{Z}}_{t-1} + \mathbf{a}_t \quad \text{.....(3.76)}$$

For *m*=2 the vector AR(2) becomes:

$$\begin{bmatrix} \dot{\mathbf{Z}}_{1,t} \\ \dot{\mathbf{Z}}_{2,t} \end{bmatrix} - \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{Z}}_{1,t-1} \\ \dot{Z}_{2,t-1} \end{bmatrix} = \begin{bmatrix} \mathbf{a}_{1,t} \\ \mathbf{a}_{2,t} \end{bmatrix}. \quad \text{.....(3.77)}$$

Note that each $\dot{\mathbf{Z}}_{i,t}$ involves not only lagged values of $\dot{\mathbf{Z}}_{i,t}$ but also lagged values of other variables $\dot{\mathbf{Z}}_{j,t}$.

If $\phi_{12}=0$, then the equation (3.68) becomes

$$\begin{bmatrix} 1-\phi_{11}B & 0 \\ 1-\phi_{21}B & 1-\phi_{22}B \end{bmatrix}\begin{bmatrix} \dot{\mathbf{Z}}_{1,t} \\ \dot{\mathbf{Z}}_{2,t} \end{bmatrix}=\begin{bmatrix} \mathbf{a}_{1,t} \\ \mathbf{a}_{2,t} \end{bmatrix} \quad \text{.....(3.78)}$$

or

$$\begin{cases} \dot{\mathbf{Z}}_{1,t}=\dfrac{1}{1-\phi_{11}B}\mathbf{a}_{1.t} \\ \dot{\mathbf{Z}}_{2,t}=\dfrac{\phi_{21}B}{(1-\phi_{22}B)}\dot{Z}_{1,t}+\dfrac{1}{1-\phi_{22}B}\mathbf{a}_{2,t} \end{cases} \quad \text{.....(3.79)}$$

To reduce equation (3.79) to a casual transfer function model, let

$$\begin{cases} \mathbf{a}_{1,t}=b_{1,t} \\ \mathbf{a}_{2,t}=\alpha a_{1,t}+b_{2,t} \end{cases}$$

where α is the regression coefficient of $\mathbf{a}_{2,t}$ on $\mathbf{a}_{1,t}$. The error term $b_{2,t}$ is independent of $\mathbf{a}_{1,t}$ and hence $b_{1,t}$.

3.3.3.1.b Vector AR(*p*) models

The general vector AR(*p*) process is given by:

$$(1-\Phi_1 B-\cdots-\Phi_p B^p)\dot{\mathbf{Z}}_t=\mathbf{a}_t \quad \text{.....(3.80)}$$

or

$$\dot{Z}_t=\Phi_1 Z_{t-1}+\cdots+\Phi_p \dot{\mathbf{Z}}_{t-p}+\mathbf{a}_t \quad \text{.....(3.81)}$$

and it is invertible. For the process to be stationary require that the zeros of $\left|\mathbf{I}-\Phi_1 B-\cdots-\Phi_p B^p\right|$ lie outside the unit root circle, or equivalently the unit root of $\left|\lambda^p I-\lambda^{p-1}\Phi_p-\cdots-\Phi_p\right|=0$ be inside the unit circle.

3.3.3.1.c Vector MA(1) models

The vector MA(1) models are given by:

$$\dot{\mathbf{Z}}_t = (1 - \Theta_1 \mathbf{B})\mathbf{a}_t$$

where the $\mathbf{a}_t$ are $m \times 1$vector white noise with mean zero and variance matrix $\sum$. For m = 2, MA(1) models can be written as

$$\begin{bmatrix} \dot{\mathbf{Z}}_{1,t} \\ \dot{\mathbf{Z}}_{2,t} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a_{1,t} \\ a_{2,t} \end{bmatrix} - \begin{bmatrix} \Theta_{11} & \Theta_{12} \\ \Theta_{21} & \Theta_{22} \end{bmatrix} \begin{bmatrix} a_{1,t-1} \\ a_{2,t-2} \end{bmatrix}. \quad \text{.....(3.82)}$$

The covariance matrix function of $\dot{\mathbf{Z}}_t$ is

$$\Gamma(0) = E\left(\dot{\mathbf{Z}}_t, \dot{\mathbf{Z}}_t'\right) = E\left[(\mathbf{I} - \Theta_1 B)\mathbf{a}_t\right]\left[(\mathbf{I} - \Theta_1 B)\mathbf{a}_t\right]'$$

$$= \sum + \Theta_1 \sum \Theta_1' \quad \text{.....(3.83)}$$

$$\Gamma(k) = E\left(\dot{\mathbf{Z}}_t, \dot{\mathbf{Z}}_{t+k}'\right) = E\left[\mathbf{a}_t - \Theta_1 \mathbf{a}_{t-1}\right]\left[\mathbf{a}_{t+k}' - \mathbf{a}_{t+k-1}\Theta_1'\right]$$

$$= \begin{cases} -\sum \Theta_1', & k = 1, \\ -\Theta_1 \sum, & k = -1 \\ 0, & |k| > 1. \end{cases}, \quad \text{.....(3.84)}$$

The $\Gamma(-1) = \Gamma(1)'$, and $\Gamma(k)$ cuts off after lag 1, a behavior that parallels to univariate MA(1) process.

3.3.3.1.d Vector MA(*q*) models

The general vector MA(*q*) process is given by:

$$\dot{\mathbf{Z}}_t = \left(\mathbf{I} - \Theta_1 B - \cdots - \Theta_q B^q\right)\mathbf{a}_t \quad \text{.....(3.85)}$$

The covariance matrix function is

$$\Gamma(k) = E\left[\mathbf{a}_t - \Theta_1 \mathbf{a}_{t-1} - \cdots - \Theta_q \mathbf{a}_{t-q}\right]\left[\mathbf{a}_{t+k}' - \mathbf{a}_{t+k-1}'\Theta_1' - \cdots - \mathbf{a}_{t+k-q}'\Theta_q'\right]$$

$$= \begin{cases} \sum_{j=0}^{q-k} \Theta_j \sum \Theta_{j+k}' & \text{for } k = 0, 1, \ldots, q, \\ 0, & k > q, \end{cases} \quad \text{.....(3.86)}$$

where $\Theta_1 = \mathbf{I}$ and $\Gamma(-k) = \Gamma'(k)$. $\Gamma(k)$ cuts off after lag *q*. The process is always stationary.

3.3.3.1.e Vector ARMA(1,1) models

The vector ARMA(1,1) is given by

$$\left(\mathbf{I}-\Phi_1 B\right)\dot{\mathbf{Z}}_t=\left(1-\Theta_1 B\right)\mathbf{a}_t. \quad \text{.....(3.87)}$$

The model is stationary if the zeros of the determinant polynomial $\left|\mathbf{I}-\Phi_1 B\right|$ are outside the unit circle or if all the eigenvalues of Φ_1 are inside the unit circle. Then the formula is given by:

$$\dot{\mathbf{Z}}_t=\sum_{s=0}^{\infty}\Psi_s\mathbf{a}_{t-s} \quad \text{.....(3.88)}$$

where the Ψ_s weights are obtained by equating the coefficients of B^j in the following matrix equation

$$\left(\mathbf{I}-\Phi_1 B\right)\left(\mathbf{I}+\Psi_1 B+\Psi_2 B^2\right)=\left(\mathbf{I}-\Theta_1 B\right) \quad \text{.....(3.89)}$$

i.e.,

$$\Psi_j=\Phi_1\Psi_{j-1}=\Phi_1^{\,j-1}\left(\Phi_1-\Theta_1\right),\quad j\geq 1.$$

The process is invertible if the zeros of $\left|\mathbf{I}-\Phi_1 B\right|$ are outside the unit circle or if all the eigenvalues of Θ_1 are inside the unit circle.

3.3.3.2 Identification of vector time series models

For a given observed vector time series $\mathbf{Z}_1,\mathbf{Z}_2,\ldots,\mathbf{Z}_n$, we identify its underlying model from the pattern of its sample correlation and partial correlation matrices after proper transformations are applied to reduce a non-stationary series to be stationary.

3.3.3.2.a Sample correlation matrix function

Given a vector time series of *n* observations $\mathbf{Z}_1,\mathbf{Z}_2,\ldots,\text{and }\mathbf{Z}_n$, the sample correlation matrix can be written as

$$\hat{\rho}(k)=\left[\hat{\rho}_{ij}(k)\right] \quad \text{.....(3.90)}$$

where $\hat{\rho}_{ij}(k)$ are the sample cross-correlations for i^{th} and j^{th} components series

$$\hat{\rho}_{ij}(k) = \frac{\sum_{t=1}^{n-k}\left(Z_{i,t} - \bar{Z}_i\right)\left(Z_{j,t+k} - \bar{Z}_t\right)}{\sqrt{\left[\sum_{t=1}^{n}\left(Z_{i,t} - \bar{Z}_i\right)^2 \sum_{t=1}^{n}\left(Z_{j,t} - \bar{Z}_j\right)^2\right]}} \quad \text{.....(3.91)}$$

where $\bar{Z}_i$ and $\bar{Z}_j$ are the sample means of the corresponding components series.

The sample correlation matrix function is useful in identifying a finite order MA model as the correlation matrices are zero beyond lag *q* for the vector MA(*q*) process.

3.3.3.2.b Partial autoregression matrices

PACF is a useful tool for identifying the order of univariate AR model. Partial autoregression function is the correlation between Z_t and Z_{t+k} after their mutual linear independency on the intervening variables $Z_{t+1}, Z_{t+2}, \ldots,$ and Z_{t+k-1} has been removed, i.e.,

$$\phi_{kk} = \frac{\mathrm{Cov}\left|\left(Z_t - \hat{Z}_t\right), \left(Z_{t+k} - \hat{Z}_{t+k}\right)\right|}{\sqrt{\mathrm{Var}\left(Z_t - \hat{Z}_t\right)}\ \sqrt{\mathrm{Var}\left(Z_{t+k} - \hat{Z}_{t+k}\right)}} \quad \text{.....(3.92)}$$

where $\hat{Z}_t$ and $\hat{Z}_{t+k}$ are the minimum mean squared error linear regression estimators of Z_t and Z_{t+k} based on $Z_{t+1}, Z_{t+2}, \ldots,$ and Z_{t+k-1} respectively. PACF is also useful in identifying the order of the univariate AR(*p*) model since ϕ_{kk} is zero for $|k| > p$.

3.4 Neural networks models

The aim of this section is to illustrate the identification, estimation and evaluation of neural network models. These models define linear relationships between a time series observation at time t, the dependent variable and a set of time series observations that occurred prior to time t.

Any linear model can be expressed as a simple feed forward neural network model with linear activation functions. The versatility of a neural network lies in the fact that it is used to model non-linear relationships between input and output variables.

3.4.1 Estimation

In neural terminology, estimation of the parameter of the model takes place during the training period, during which the network learns with generalisation through certain learning

algorithms. Weights connected to each input value (parameters) are constantly updated with the error function, $E(w)$, which is often the sum of squares errors that reaches its global minimum.

3.4.2 Evaluation

After a model has been identified and the parameters estimated, the model must be evaluated. Evaluation procedures are the same for both ARIMA and neural network model. If the model fits the data well, then the residuals should almost have the properties of uncorrelated, identically distributed, random variables with mean and fixed standard deviation (Cryer, 1986). If the estimated value of Y_t is given by

$$\hat{Y}_t = f(Y_{t-1}, Y_{t-2}, \cdots, Y_{t-p}, \hat{w}); \; t = 1, 2, \cdots, N. \quad \text{.....(3.93)}$$

Then the residual at time t is defined as $\hat{\varepsilon} = Y_t - \hat{Y}_t; \; t = 1, 2, \cdots, N$

The residuals of a fitted model are useful indicators of any inadequacies in the specification of the model or violations of underlying assumptions. Examination of various plots of the residuals is an indispensable step in the evaluation process of any model (Box and Jenkins). If a plot of the residuals exhibits a trend over time, it is an indication that the trend in the data is not adequately modeled.

A histogram of standardised residuals should correspond with symmetrical normal curve if the model fits the data well. Any outliers will imply significant differences between the $\hat{Y}_t$ and the corresponding observed value, Y_t that should be investigated since the model fits the data poorly at those points. The sample autocorrelation function of the residuals $\hat{\gamma}_k$, can be observed to check for the independence of the residual in the model. Usually the sample autocorrelations are approximately uncorrelated and normally distributed with mean of zero and constant variance.

3.4.3 Forecasting a time series using Neural Network

In this chapter a neural network model is used to forecast a time series. Bootstrap methods can be used to calculate standard errors and prediction limits for the forecasts. Suppose that Y_t, $t = \cdots -1, 0, 1, \cdots$ is an equally spaced weakly stationary or covariance stationary time

series. A linear model of the analysis of time series in the domain belongs to an ARMA class of the form:

$$Y_t = \theta_0 + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_p Y_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \cdots + \theta_q \varepsilon_{t-q} \quad \text{.....(3.94)}$$

where $\{\varepsilon_t\}$ is a sequence of uncorrelated variables, also as a white noise process, with conditions $E\{\varepsilon_t\} = 0$ and $E\{\varepsilon_t, \varepsilon_t\} = \begin{cases} \sigma_a^2 & \text{for } t = \tau \\ 0 & \text{otherwise} \end{cases}$

and $\theta_0, \phi_1, \cdots, \phi_p, \theta_t, \cdots \theta_q$ are unknown constants or parameters.

Considering the ARMA model (equation (3.94)) for a stationary time series $\{Y_t\}$ with $t = 1, 2, \cdots, N$. The linear AR(1) model is given by

$$Y_t = \phi_t Y_{t-1} + \varepsilon_t \quad \text{.....(3.95)}$$

where $\{\varepsilon_t\}$ is a white noise process.

By using equation (3.95) for the second step predictive, Y_{N+2} ; the MSE linear predictor can be written in terms of the observed data; Y_N :

$$Y_N(2) = \phi_1 E\left(Y_{N+1} | Y_1, Y_{2,\cdots,} Y_N\right) + 0 = \phi Y_N(1) - \phi_1 Y_N \quad \text{.....(3.96)}$$

In case of a nonlinear model such as the neural network;

$$Y_t = f\left(Y_{t-1}, a\right) + \varepsilon_i \quad \text{.....(3.97)}$$

The one step minimum MSE predictor is given by

$$Y_N(1) = E\left(Y_{N+1} | Y_1, Y_{2,\cdots,} Y_N\right) = f(Y_N, \alpha) \quad \text{.....(3.98)}$$

Two or more step forecasts are expectations of non-linear functions, for example

$$\begin{aligned} Y_N(2) &= E[\, f(Y_{N+1}, \alpha) + \varepsilon_{N+1} | Y_1, Y_2, \cdots, Y_N \,] \\ &= E\{F[[\, f(Y_N, \alpha) + \varepsilon_{N+1}], \alpha] | Y_1, Y_2, \cdots, Y_N\} \quad \text{.....(3.99)} \\ &= E\{F[[\, f(Y_N(1) + \varepsilon_{N+1}], \alpha] | Y_1, Y_2, \cdots, Y_N\} \end{aligned}$$

Multi-step forecasts for neural network models

Let $Y_1, Y_2, \cdots, Y_N$ be a stationary time series described by

$$Y_t = f\left(Y_{t-1}, Y_{t-2}, \cdots, Y_{t-p}; \underline{w}\right) + \varepsilon_t \quad \text{.....(3.100)}$$

Where $f()$ is a non-linear function defined by the neural architecture, p is the number of input variables in the network, and $\underline{w}$ the weight vector.

The $\varepsilon_t's$ are uncorrelated, identically distributed random variables with mean zero and variance. The observation at time $N+1$ can be written as

$$Y_{N+1} = F(Y_N, Y_{N-1}, \ldots, Y_{N+1-p}, \underline{w}) + \varepsilon_{t+1}. \quad \text{.....(3.101)}$$

The minimum MSE forecast for a single step is

$$Y_N(1) = f\left(Y_N, Y_{N-1}, \ldots, Y_{N+1-p}; \underline{w}\right) \text{ with estimator } \quad \text{.....(3.102)}$$

$$\hat{Y}_N(1) = f\left(Y_N, Y_{N-1}, \ldots, Y_{N+1-p}; \underline{\hat{w}}\right) \quad \text{.....(3.103)}$$

The observation at time $N+2$ is $Y_{N+2} = F(Y_{N+1}, Y_N, \cdots, Y_{N+2-p}, \underline{w}) + \varepsilon_{t+2}$ with the minimum MSE forecast

$$\hat{Y}_N(2) = [\, f\left(Y_{N+1}, Y_N, \ldots, Y_{N+2-p}; \underline{w}\right) | Y_1, Y_2, \cdots, Y_N \,] \quad \text{.....(3.104)}$$

For the estimation of the MSE forecast, the bootstrap method can be used and therefore the bootstrap forecast for $Y_N(2)$ is proposed as

$$\hat{Y}_N(2) = \frac{1}{m} \sum_{j=1}^{m} f(Y_{N+1}^{*(j)}, Y_N, \ldots, Y_{N+2-p}; \underline{\hat{w}}) \quad \text{.....(3.105)}$$

where

$$Y_{N+1}^{*(j)} = f(Y_N, Y_{N-1}, \ldots, Y_{N+1-p}; \underline{\hat{w}}) + \varepsilon_{N+1}^{*(j)}$$

And $\varepsilon_{N+1}^{*(j)}$ is an observation, drawn with replacement, from $\hat{\varepsilon}_{p+1}, \ldots, \hat{\varepsilon}_N$ with

$$\hat{\varepsilon}_t = Y_t - f(Y_{t-1},\ldots,Y_{t-p},\underline{\hat{w}}) \text{ with } t = p+1, p+2,\cdots,N. \quad \text{.....(3.106)}$$

For a forecast of h time steps, Y_{N+h}, the estimator is

$$\hat{Y}_N(h) = \frac{1}{m}\sum_{j=1}^{m} f(Y^{*(j)}_{N+h-1}, Y^{*(j)}_{N+h-2},\ldots,Y^{*(j)}_{N+h-p};\underline{\hat{w}}) \quad \text{.....(3.107)}$$

where

$$Y^{*(j)}_{N+h} = f(Y^{*(j)}_{N+h-1}, Y^{*(j)}_{N+h-2},\ldots,Y^{*(j)}_{N+h-p};\underline{\hat{w}}) + \varepsilon^{*(j)}_{N+h}$$

and

$$Y^{*(j)}_{N+h-p} = Y_{N+h-p} \text{ if } h-p \le 0$$

The bootstrap procedure can be summarised in few steps: First fit the model (using equation (3.100)) to the time series $Y_1, Y_2,\ldots,Y_N$. Then calculate estimates of the residual term using equation (3.105). The last step is to calculate $Y^*_{N+1},\ldots,Y^*_{N+h}$ condition on $Y_1, Y_2,\ldots,Y_N$.

$$Y^*_{N+h} = f(Y^*_{N+h-1}, Y^*_{N+h-2},\ldots,Y^*_{N+h-p};\underline{\hat{w}}) + \varepsilon^*_{N+h} \quad \text{.....(3.108)}$$

where

$Y^*_{N+h-p} = Y_{N+h-p}$ if $h-p \le 0$ and ε^*_{N+h} is an observation drawn randomly with replacement from $\hat{\varepsilon}_{p+1},\ldots,\hat{\varepsilon}_N$. Usually repeat this m times where $m = 100$. Now calculate the one-to step forecasts the time series generated in the previous step using equations (3.102), (3.105) and (3.107).

The prediction limits for Y_{N+h}

The prediction error is

$$e_N(h) = Y_{N+h} - \hat{Y}_N(h)$$

and the standardised prediction error is defined as:

$$r_N(h) = \frac{e_N(h)}{\hat{\sigma}}.$$

By using the bootstrap methodology the distribution of $r_N(h)$ can be approximated using the Monte Carlo algorithm.

An approximate $1-\gamma$ prediction interval for Y_{N+h} is

$$[\hat{Y}_N(h) + r_L\hat{\sigma}; \hat{Y}_N(h) + r_U\hat{\sigma}]$$

where

r_L and r_U are the $B\left(\frac{\gamma}{2}\right) - \text{th}$ and $B\left(1-\frac{\gamma}{2}\right) - \text{th}$ order statistic of $r_N(h)_1^*, \ldots, r_N(h)_B^*$, respectively, and *B* is the number of bootstrap replications. To estimate the standard error of $e_N(h), \sigma,$ a large number of bootstrap realisations of $e_N(h)$ is required; $r_N(h)$ is a function of $\hat{\sigma}$. For good approximation of the distribution of $r_N(h)$, at least 1000 bootstrap replications are required.

3.5 Chapter summary

In chapter 3 the research methodology to be adopted in this study has been defined.

In the next chapter the data used for the study is analysed and interpreted.

CHAPTER 4: DATA ANALYSIS AND RESULTS

This chapter presents results from Minitab, Stats Graphics, Zaintun Time Series, SAS and SPSS on the analysis of Stats SA data on “Manufacturing: Production and Sales” for the period January 1998 to December 2010. The results generated by the various statistical software packages are included in Appendix A.

The results presented in this chapter are based on the following five variables pertaining to electrical appliances included in the data set:

1. Lighting equipment
2. Electric machines
3. Other electrical equipment
4. Communication apparatus
5. Accumulators

The analysis is performed using the following statistical techniques:

a. Univariate time series analysis (ARIMA models and forecasting)
b. Multivariate time series analysis
c. Regression analysis

In section 4.1 we analyses the ARIMA models. Section 4.2 analyses ES and differences between three types of ES while section 4.3 studies the regression analyses. Finally, section 4.4 analyses the multivariate time series.

4.1 ARIMA models

The ARIMA (autoregressive integrated moving average) procedure analyses and forecasts equally spaced univariate time series data, and intervention data using autoregressive integrated moving average. The model predicts a value in a response time series as a linear combination of its own past values, current and past values of other time series. ARIMA models consist of three steps. The first step is model identification in which the observed series is transformed (differenced) to be stationary. The second step is model estimation, in which the orders *p* and *q* are selected and corresponding parameters are estimated. The third

and final step is forecasting, in which the estimated model is then used to forecast future values of the observed time series.

4.1.1 Lighting equipment

Figure 4.1 shows that a time series plot of differenced/transformed data for lighting equipment is stationary for a regular pattern since the mean is zero but still strongly seasonal since lighting peaks repeat after fixed time intervals. The accuracy measures (MAPE, MAD and MSD) are computed by Minitab. All measures are based on the 'errors' (deviations) between the actual and fitted values. Montgomery *et al*. (2008) indicated that the accuracy measures are used to compare different methods of modeling time series. The smaller the value of the accuracy measure the better the fit of the model.

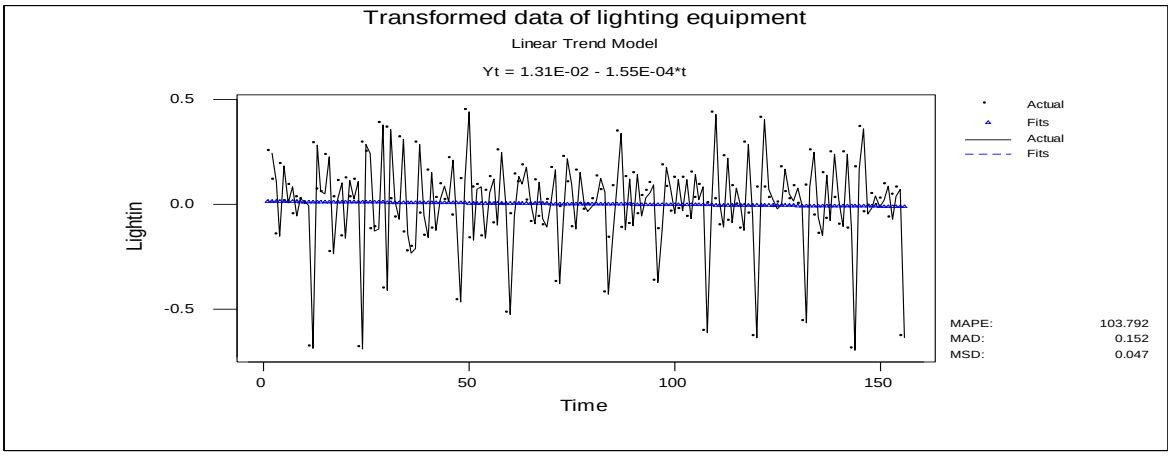
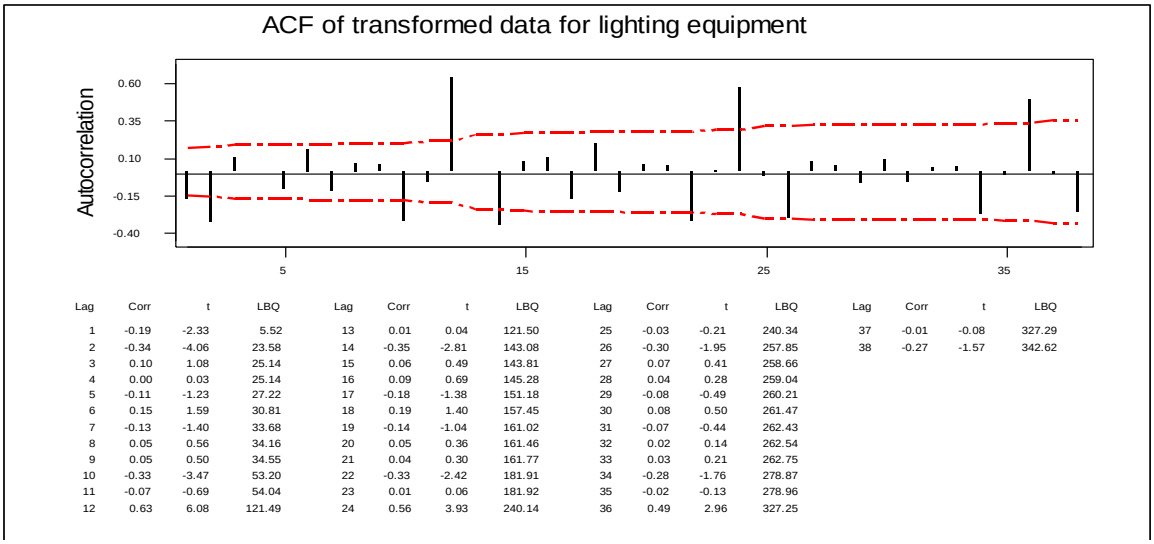


Figure 4.1 Trend of transformed data for lighting equipment

Figure 4.2 and figure 4.3 show an ACF and PACF of transformed data of lighting equipment, respectively. If all of the bars fall within the indicated confidence intervals (the dotted lines), there are no significant autocorrelations in the series. On the other hand, if the bars cross the dotted lines then there are correlations in the series.

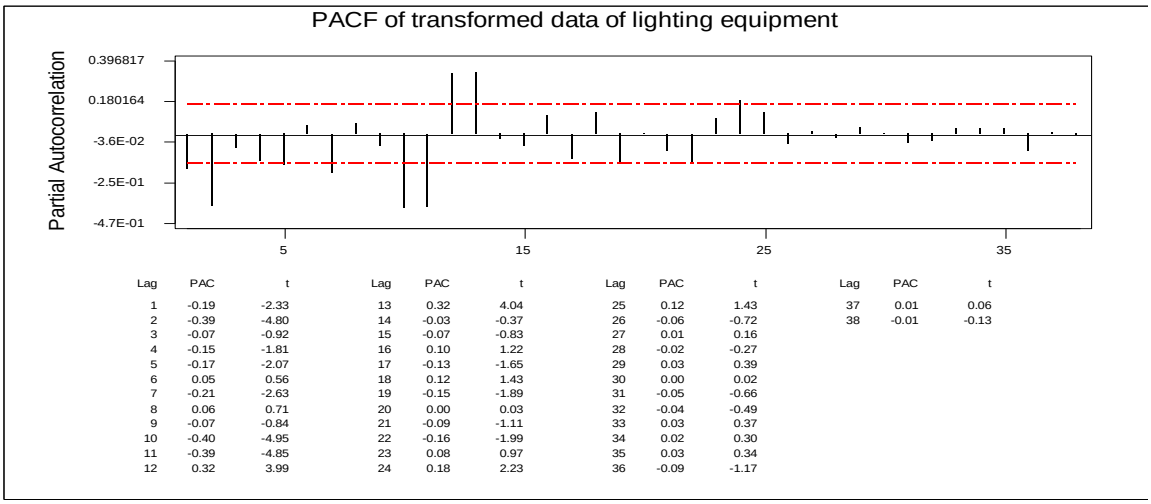
Figure 4.2 confirms that very little autocorrelation remains in the series after differencing has been applied. This plot also suggests that a simple model which incorporates seasons of lag 12, lag 24 and lag 36 autocorrelations must be adequate seasons.

The PACF displayed in figure 4.3 also confirms that very little autocorrelation still remains in the series after differencing at lag 2, and between lag 10 and lag 14, inclusively.



Lag	Corr	t	LBQ	Lag	Corr	t	LBQ	Lag	Corr	t	LBQ	Lag	Corr	t	LBQ
1	-0.19	-2.33	5.52	13	0.01	0.04	121.50	25	-0.03	-0.21	240.34	37	-0.01	-0.08	327.29
2	-0.34	-4.06	23.58	14	-0.35	-2.81	143.08	26	-0.30	-1.95	257.85	38	-0.27	-1.57	342.62
3	0.10	1.08	25.14	15	0.06	0.49	143.81	27	0.07	0.41	258.66				
4	0.00	0.03	25.14	16	0.09	0.69	145.28	28	0.04	0.28	259.04				
5	-0.11	-1.23	27.22	17	-0.18	-1.38	151.18	29	-0.08	-0.49	260.21				
6	0.15	1.59	30.81	18	0.19	1.40	157.45	30	0.08	0.50	261.47				
7	-0.13	-1.40	33.68	19	-0.14	-1.04	161.02	31	-0.07	-0.44	262.43				
8	0.05	0.56	34.16	20	0.05	0.36	161.46	32	0.02	0.14	262.54				
9	0.05	0.50	34.55	21	0.04	0.30	161.77	33	0.03	0.21	262.75				
10	-0.33	-3.47	53.20	22	-0.33	-2.42	181.91	34	-0.28	-1.76	278.87				
11	-0.07	-0.69	54.04	23	0.01	0.06	181.92	35	-0.02	-0.13	278.96				
12	0.63	6.08	121.49	24	0.56	3.93	240.14	36	0.49	2.96	327.25				

Figure 4.2 The ACF of transformed data for lighting equipment



Lag	PAC	t	Lag	PAC	t	Lag	PAC	t	Lag	PAC	t
1	-0.19	-2.33	13	0.32	4.04	25	0.12	1.43	37	0.01	0.06
2	-0.39	-4.80	14	-0.03	-0.37	26	-0.06	-0.72	38	-0.01	-0.13
3	-0.07	-0.92	15	-0.07	-0.83	27	0.01	0.16			
4	-0.15	-1.81	16	0.10	1.22	28	-0.02	-0.27			
5	-0.17	-2.07	17	-0.13	-1.65	29	0.03	0.39			
6	0.05	0.56	18	0.12	1.43	30	0.00	0.02			
7	-0.21	-2.63	19	-0.15	-1.89	31	-0.05	-0.66			
8	0.06	0.71	20	0.00	0.03	32	-0.04	-0.49			
9	-0.07	-0.84	21	-0.09	-1.11	33	0.03	0.37			
10	-0.40	-4.95	22	-0.16	-1.99	34	0.02	0.30			
11	-0.39	-4.85	23	0.08	0.97	35	0.03	0.34			
12	0.32	3.99	24	0.18	2.23	36	-0.09	-1.17			

Figure 4.3 The PACF of transformed data for lighting equipment

4.1.1.1 Outlier identification: Lighting equipment using Grubbs' test

Test statistic = 2.9392

P-Value = 0.45432

The Grubbs' test is used to detect outliers from normal distributions. The test detects one outlier at a time and if the outlier is found (i.e. it is above 3.5 standard deviation) it is removed from the data and the test repeated until no outliers are detected.

Grubbs' test analysis identifies and treats potential outliers in samples from normal populations. Tables 4.1 to 4.3 display the usual estimates of the mean and standard deviation together with estimates which are designed to be resistant to outliers. For the 156 values of lighting equipment, the sample mean and sample standard deviation are 107.851 and 26.078, respectively. The corresponding Winsorized estimates of which 15.0% of the largest and

smallest data values are replaced by values from the interior of the sample, are 106.364 and 27.106.

Table 4.1 Location estimates of lighting equipment

Sample mean	107.851
Sample median	105.650
Trimmed mean	105.834
Winsorized mean	106.364

Trimming: 15.0%

Table 4.2 Scale estimates of lighting equipment

Sample std. deviation	26.0781
MAD/0.6745	25.9451
Sbi	25.8022
Winsorized sigma	27.1055

Table 4.3 The 95% confidence interval for the mean of lighting equipment

	Lower Limit	Upper Limit
Standard	103.727	111.976
Winsorized	101.242	111.486

Table 4.4 shows the smallest and largest values of lighting equipment. The Studentised values measure how many standard deviations each value is from the sample mean of 107.851. The most extreme value is in row 142, which is 2.9392 standard deviations from the mean. Since the p-value for Grubbs' test is greater or equal to 0.05, 184.5 is not a significant outlier at the 5% significance level, assuming that all the other values follow a normal distribution. Similar scores are displayed after deleting each point one at a time when computing the sample statistics, and when the mean and standard deviation are based on the MAD. Values of the modified scores greater than 3.5 in absolute value, will be outliers of which in this case there is none as indicated by table 4.4 and figure 4.4.

Table 4.4 Sorted Values of lighting equipment

Row	Value	Studentised Values Without Deletion	Studentised Values With Deletion	Modified MAD Z-Score
24	53.1	-2.09951	-2.13704	-2.02543
12	57.6	-1.92695	-1.95685	-1.85198
48	65.2	-1.63552	-1.65519	-1.55906
30	68.7	-1.50131	-1.51726	-1.42416
85	70.0	-1.45146	-1.46617	-1.37405
...				
128	165.7	2.21829	2.26181	2.31450
129	168.5	2.32566	2.37518	2.42240
131	181.0	2.80499	2.88872	2.90420
130	182.1	2.84717	2.93456	2.94660
142	184.5	2.93920	3.03496	3.03910

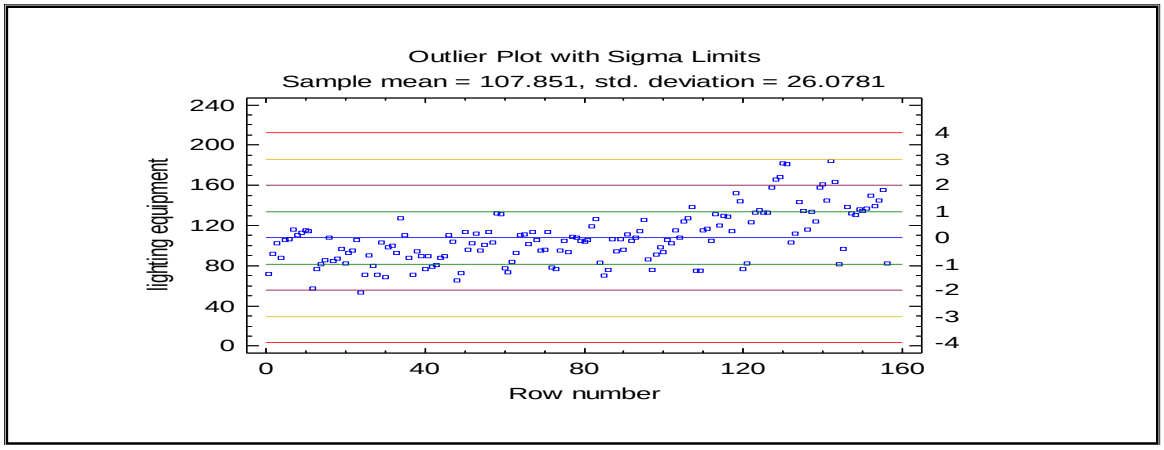


Figure 4.4 Outlier plot with sigma limits

4.1.1.2 Model comparison for lighting equipment

Stats Graphics was used, and automatically suggested a set of models that can be fitted. On the basis of these models, the Akaike Information Criterion (AIC) method is used to select the best one. The model with the minimum value of the AIC is considered as the best model. Models which are closer to the minimum AIC are acceptable.

Table 4.5 compares the results of fitting different models to the data.

The model with the lowest value of the AIC is model M, which has been used to generate the forecasts.

(A) Random walk
(B) Constant mean = 107.748
(C) Linear trend = 81.7286 + 0.331462 t
(M) ARIMA(0,1,1)x$(1,0,2)_{12}$
(N) ARIMA(0,1,1)x$(1,0,1)_{12}$
(O) ARIMA(0,1,1)x$(2,0,1)_{12}$
(P) ARIMA(1,1,1)x$(1,0,2)_{12}$
(Q) ARIMA(1,1,1)x$(2,0,1)_{12}$

Table 4.5 summarises the performance of the currently selected model M in fitting the historical data. The results display the following:

(1) the root mean squared error (RMSE)
(2) the mean absolute error (MAE)
(3) the mean absolute percentage error (MAPE)
(4) the mean error (ME)
(5) the mean percentage error (MPE)

Each of the above statistics is based on the one-ahead forecast errors. The first three statistics measure the magnitude of the errors (RMSE, MAE and MAPE). A better model will give a smaller value of the AIC. The last two statistics (MPE and ME) measure bias. A better model will give a value of AIC close to zero.

Table 4.5 Model comparison for lighting equipment

Model	RMSE	MAE	MAPE	ME	MPE	AIC
(A)	12.02930	9.56067	9.37368	0.029027	-0.680228	5.10891
(B)	21.10230	15.87450	14.70720	0.102889	-3.238700	6.25261
(C)	13.55270	10.30170	9.78697	0.043471	-1.374600	5.37984
(M)	10.52020	8.18041	8.00659	0.769022	0.042480	4.75787
(N)	10.59330	8.28534	8.10267	0.784672	0.041609	4.75891
(O)	10.57970	8.27993	8.16303	-0.121928	-0.877561	4.76915
(P)	10.52210	8.20329	7.98851	0.991610	0.231558	4.77106
(Q)	10.53390	8.20324	7.98815	0.979127	0.211299	4.77330

RMSE = Root Mean Squared Error

RUNS = Test for excessive runs up and down

RUNM = Test for excessive runs above and below median

AUTO = Box-Pierce test for excessive autocorrelation

MEAN = Test for difference in mean 1st half to 2nd half

VAR = Test for difference in variance 1st half to 2nd half

OK = not significant (p >= 0.05)

* = marginally significant (0.01 < p <= 0.05)

** = significant (0.001 < p <= 0.01)

*** = highly significant (p <= 0.001)

Table 4.6 summarises the results of five tests run on the residuals to determine whether or not each model is adequate for the data. An “OK” means that the model passes the test at 95% confidence level. One *(asterisk) means that the model fails at the 95% confidence level. Two *(asterisks) mean that it fails at the 99% confidence level. Three *(asterisks) mean that the model fails at the 99.9% confidence level. The currently selected model is model M, which passes all the five (5) tests. Since no tests are statistically significant at the 95% or higher confidence level, the current model is adequate for the data.

Table 4.6 Estimation period of lighting equipment

Model	RMSE	RUNS	RUNM	AUTO	MEAN	VAR
(A)	12.0293	OK	***	***	OK	OK
(B)	21.1023	OK	***	***	***	***
(C)	13.5527	OK	***	***	OK	OK
(M)	10.5202	OK	OK	OK	OK	OK
(N)	10.5933	OK	OK	OK	OK	OK
(O)	10.5797	OK	OK	OK	OK	OK
(P)	10.5221	OK	OK	OK	OK	OK
(Q)	10.5339	OK	OK	OK	OK	OK

Figure 4.5 also confirms that the selected model M since there is no autocorrelation.

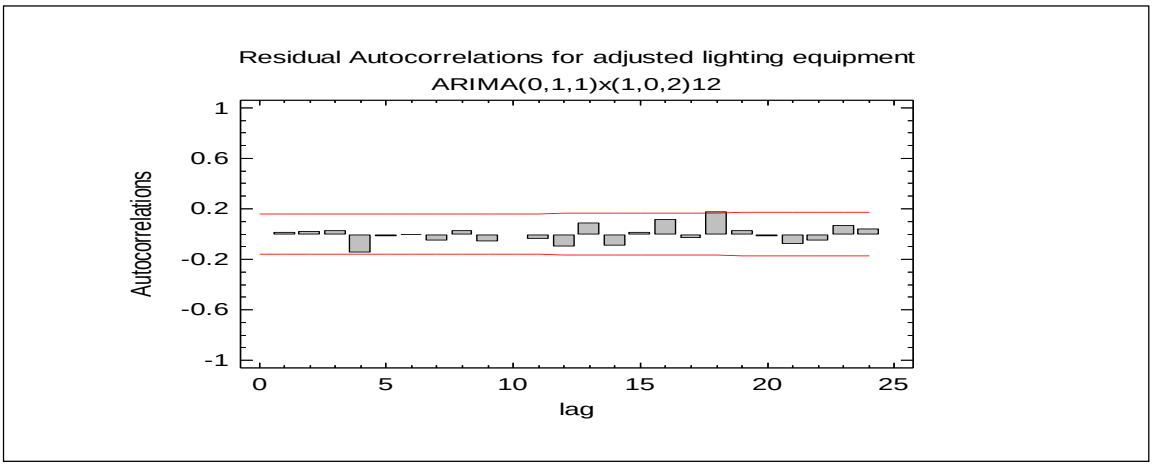


Figure 4.5 Residual autocorrelation for adjusted lighting equipment

Table 4.7 summarises the statistical significance of the terms in the forecasting model. Terms with p-values less than 0.05 are statistically significantly different from zero at 5% level of significance. The p-values for the parameters, MA(1), SAR(1) and SMA(2) are less than 0.05, and hence significantly different from zero.

Table 4.7 Estimates of parameter for lighting equipment

Parameter	Estimate	Stnd. Error	T	P-value
MA(1)	0.727775	0.0577587	12.6003	0.000000
SMA(1)	0.707191	0.0790038	8.95136	0.000000
SMA(2)	0.175834	0.0761189	2.31000	0.022350

Estimated white noise variance = 198.877 with 140 degrees of freedom
Estimated white noise standard deviation = 14.1024

4.1.1.3 Diagnostic check for lighting equipment

After a chosen model has been fitted to the data, the adequacy of the model should be examined. This is done through residuals analysis and modified Box-Pierce test. The modified Box-Pierce test is used to check if autocorrelation still exist in the residuals.

The modified Box-Pierce (Ljung-Box) test suggests that there is no autocorrelation left in the residuals. This is also confirmed by the sample ACF and the PACF of the residuals since they are both lying within the limits of residual plots shown in figure 4.6 and figure 4.7, respectively. So the model can be fitted well.

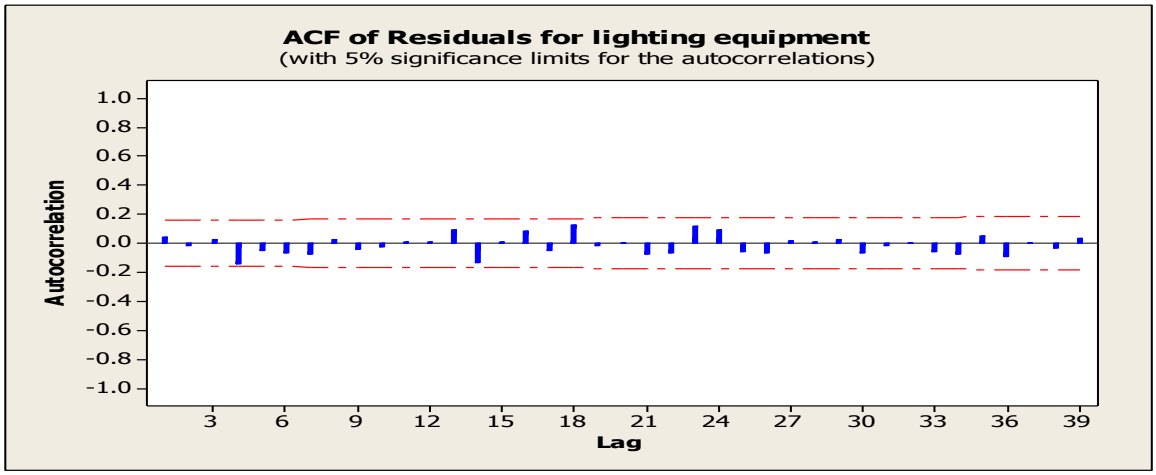


Figure 4.6 ACF of residuals for lighting equipment

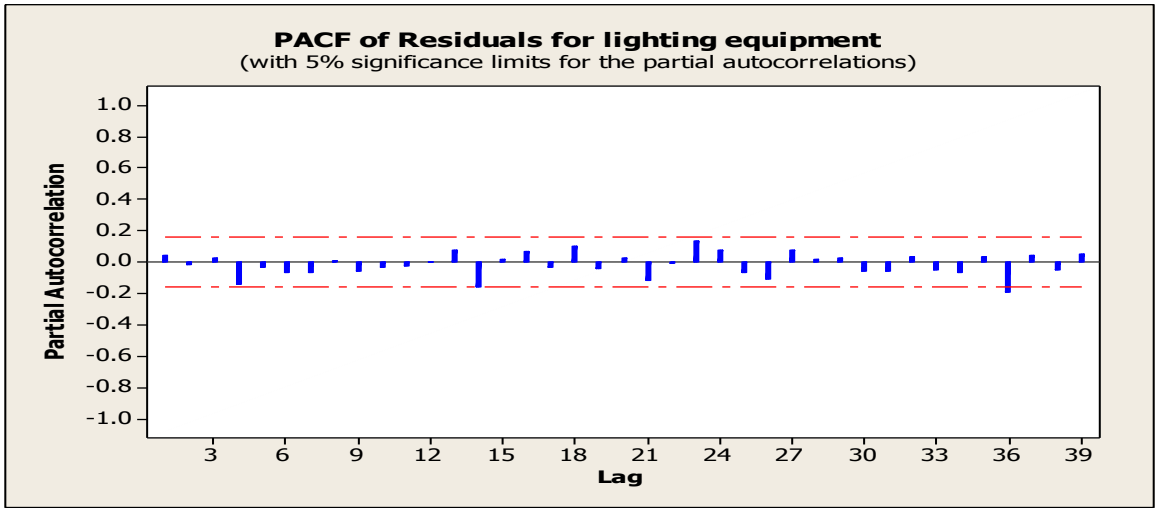


Figure 4.7 PACF of residuals for lighting equipment

Table 4.8 Modified Box-Pierce Chi-Square statistic of lighting equipment

Modified Box-Pierce (Ljung-Box) Chi-Square statistic				
Lags	12	24	36	48
Chi-Square	6.100	21.200	27.900	36.200
DF	7	19	31	43
P-Value	0.527	0.328	0.624	0.758

The last diagnostic check is the 4-in-1 residual plots for the differenced data of lighting equipment given in figure 4.8. It can be observed from the residuals against the percentage plot that the points are close to the line which assumes that the residuals are independent and normally distributed.

The fitted values and the residual plot show no pattern, which suggests that the model is adequate. The histogram of the residuals is approximately symmetric about mean zero which suggests that the residuals are normally distributed.

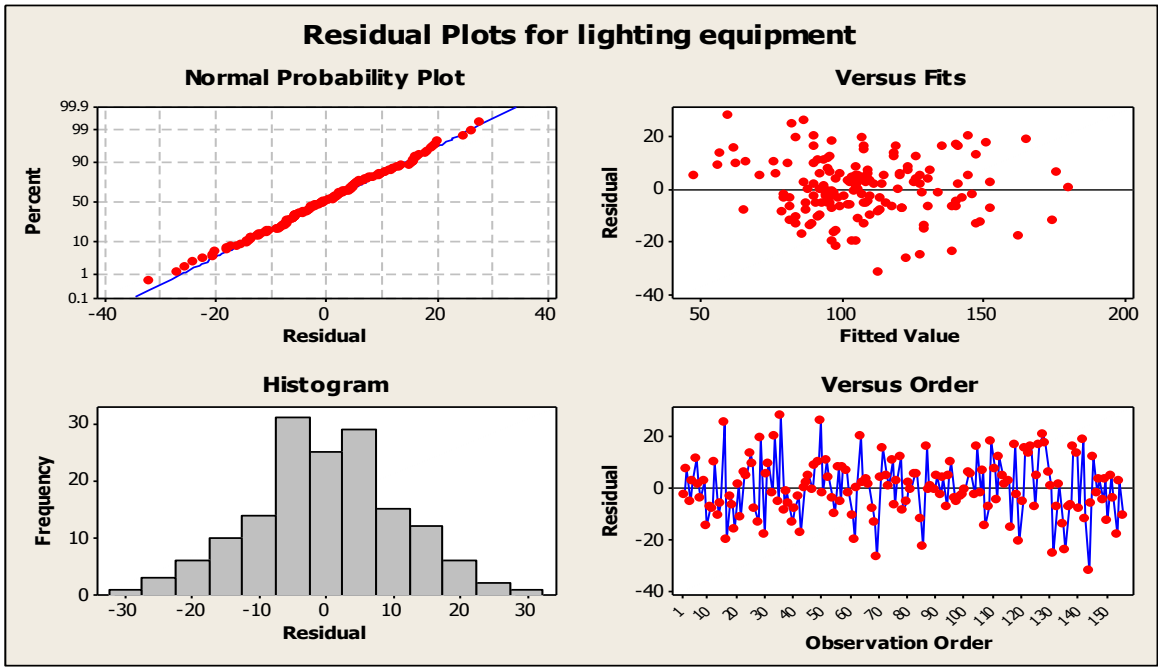


Figure 4.8 Residual plots for lighting equipment

Table 4.9 presents the forecasted values and the confidence interval for the year 2011 of lighting equipment using ARIMA (0, 1, 1)x(1, 0, 2)$_{12}$.

Table 4.9 Forecasts for lighting equipment

LL	FORECAST FOR LIGHTING EQUIPMENT (2011)	UL
70.67997	92.7884	114.8968
98.37080	122.1202	145.8696
98.50581	123.7899	149.0741
92.04348	118.7743	145.5052
101.41890	129.5221	157.6254
93.45740	122.8690	152.2807
105.93100	136.5953	167.2595
107.43690	139.3046	171.1723
101.76100	134.7883	167.8156
117.82750	151.9751	186.1226
117.55540	152.7876	188.0198
53.59378	89.8782	126.1626

4.1.2 Electric machines

A visual inspection of figure 4.9 reveals that the mean and the variance remain stable while there are some short runs where successive observations tend to follow each other for very brief durations.

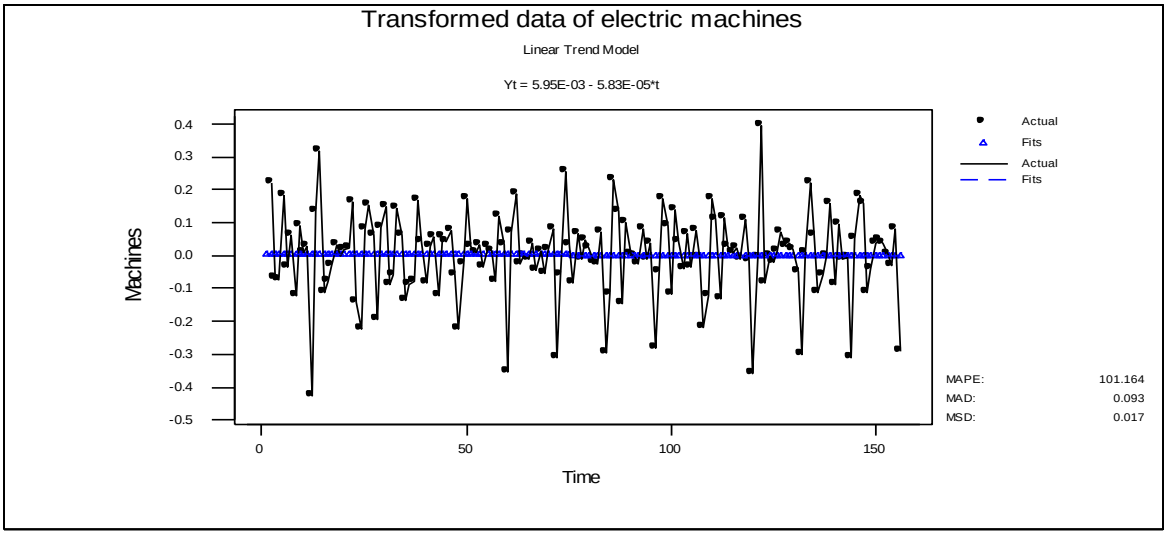
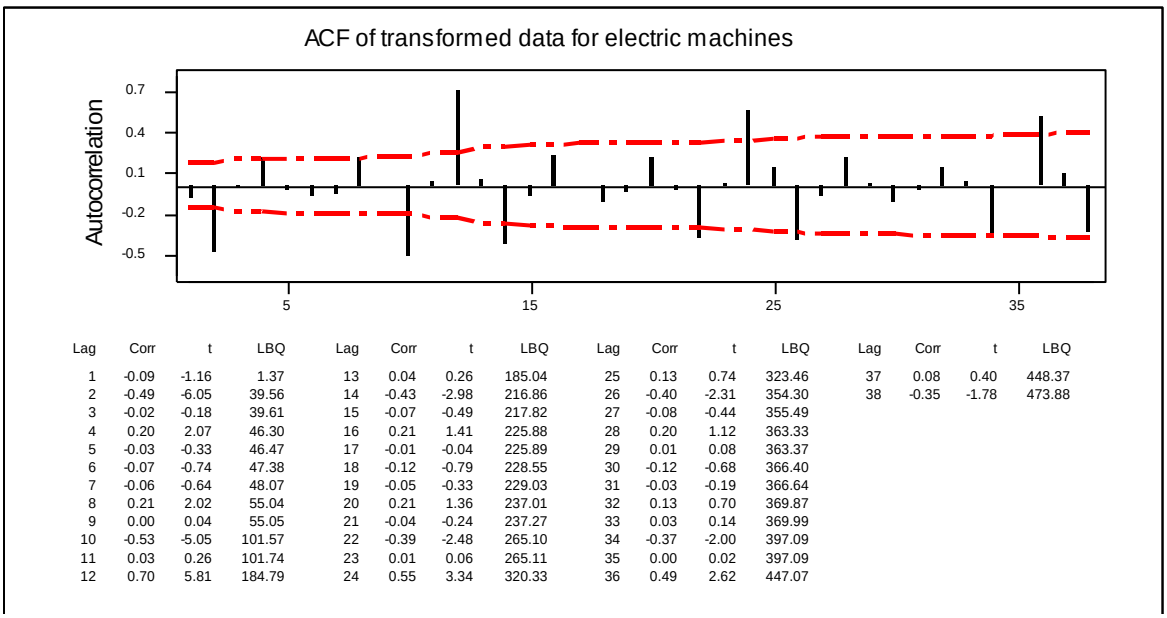


Figure 4.9 Time series plot of electric machines

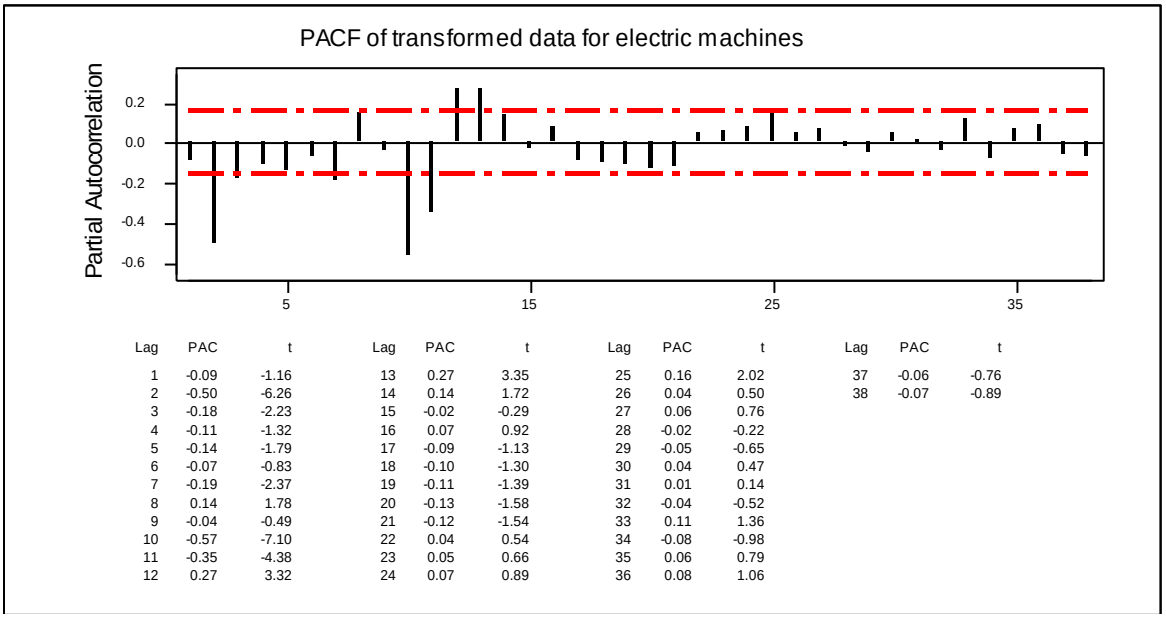
As shown from ACF plot in figure 4.10 most of the correlations are small. There are fairly large negative correlations at certain lags (i.e. lag 1, 10, 12, 14, 22 and 27) and large positive correlations of 0.7, 0.55 and 0.49 at lag 12, lag 24 and lag 36, respectively.

Figure 4.11 of the PACF shows that there are large negative correlations of -7.10, -6.26 and -4.38 at lag 10, lag 2 and lag 11, respectively and a positive correlation of 0.27 for both lag 12 and lag 13.



Lag	Corr	t	LBQ	Lag	Corr	t	LBQ	Lag	Corr	t	LBQ	Lag	Corr	t	LBQ
1	-0.09	-1.16	1.37	13	0.04	0.26	185.04	25	0.13	0.74	323.46	37	0.08	0.40	448.37
2	-0.49	-6.05	39.56	14	-0.43	-2.98	216.86	26	-0.40	-2.31	354.30	38	-0.35	-1.78	473.88
3	-0.02	-0.18	39.61	15	-0.07	-0.49	217.82	27	-0.08	-0.44	355.49				
4	0.20	2.07	46.30	16	0.21	1.41	225.88	28	0.20	1.12	363.33				
5	-0.03	-0.33	46.47	17	-0.01	-0.04	225.89	29	0.01	0.08	363.37				
6	-0.07	-0.74	47.38	18	-0.12	-0.79	228.55	30	-0.12	-0.68	366.40				
7	-0.06	-0.64	48.07	19	-0.05	-0.33	229.03	31	-0.03	-0.19	366.64				
8	0.21	2.02	55.04	20	0.21	1.36	237.01	32	0.13	0.70	369.87				
9	0.00	0.04	55.05	21	-0.04	-0.24	237.27	33	0.03	0.14	369.99				
10	-0.53	-5.05	101.57	22	-0.39	-2.48	265.10	34	-0.37	-2.00	397.09				
11	0.03	0.26	101.74	23	0.01	0.06	265.11	35	0.00	0.02	397.09				
12	0.70	5.81	184.79	24	0.55	3.34	320.33	36	0.49	2.62	447.07				

Figure 4.10 ACF of transformed data for electric machines



Lag	PAC	t	Lag	PAC	t	Lag	PAC	t	Lag	PAC	t
1	-0.09	-1.16	13	0.27	3.35	25	0.16	2.02	37	-0.06	-0.76
2	-0.50	-6.26	14	0.14	1.72	26	0.04	0.50	38	-0.07	-0.89
3	-0.18	-2.23	15	-0.02	-0.29	27	0.06	0.76			
4	-0.11	-1.32	16	0.07	0.92	28	-0.02	-0.22			
5	-0.14	-1.79	17	-0.09	-1.13	29	-0.05	-0.65			
6	-0.07	-0.83	18	-0.10	-1.30	30	0.04	0.47			
7	-0.19	-2.37	19	-0.11	-1.39	31	0.01	0.14			
8	0.14	1.78	20	-0.13	-1.58	32	-0.04	-0.52			
9	-0.04	-0.49	21	-0.12	-1.54	33	0.11	1.36			
10	-0.57	-7.10	22	0.04	0.54	34	-0.08	-0.98			
11	-0.35	-4.38	23	0.05	0.66	35	0.06	0.79			
12	0.27	3.32	24	0.07	0.89	36	0.08	1.06			

Figure 4.11 PACF of transformed data for electric machines

4.1.2.1 Outlier identification: Electric machines using Grubbs' test

Test statistic = 2.52183

P-Value = 1.0

Grubbs' test analysis identifies and treats potential outliers in samples from normal populations. Table 4.10 to table 4.12 display the usual estimates of the mean and standard deviation, together with estimates which are designed to be resistant to outliers. For the 156 values of electric machines, the sample mean and sigma are 110.613 and 14.241, respectively. The corresponding Winsorized estimates, of which 15% of the largest and smallest data values are replaced by values from the interior of the sample, are 110.443 and 14.3228 for location estimates and scale estimates, respectively.

Table 4.10 Location estimates of electric machines

Sample mean	110.613
Sample median	110.400
Trimmed mean	110.504
Winsorized mean	110.443

Trimming: 15.0%

Table 4.11 Scale estimates of electric machines

Sample std. deviation	14.2410
MAD/0.6745	12.7502
Sbi	14.3923
Winsorized sigma	14.3228

Table 4.12 The 95% confidence interval for the mean of electric machines

	Lower Limit	Upper Limit
Standard	108.361	112.866
Winsorized	107.736	113.150

Table 4.13 shows the smallest and largest values of electric machines. The Studentised values measure how many standard deviations each value is from the sample mean of 110.613. The most extreme value is the one in row 85, which is 2.52183 standard deviations from the mean. Since the p-value for Grubbs' test is greater than 0.05, that value is not a significant outlier at the 5% significance level, assuming that all the other values follow a

normal distribution. Similar scores are displayed after deleting each point one at a time when computing the sample statistics, and when the mean and standard deviation are based on the MAD. Values of the modified scores greater than 3.5 in absolute value will be considered as outliers, of which there are none.

Table 4.13 Sorted Values of electric machines

Row	Value	Studentised Values Without Deletion	Studentised Values with Deletion	Modified MAD Z-Score
85	74.7	-2.52183	-2.58381	-2.79996
12	77.4	-2.33224	-2.38215	-2.58820
73	79.3	-2.19882	-2.24132	-2.43918
97	81.2	-2.06540	-2.10132	-2.29016
84	84.0	-1.86879	-1.89639	-2.07056
...				
153	137.7	1.90201	1.93091	2.14115
129	140.5	2.09862	2.13610	2.36075
147	141.6	2.17586	2.21717	2.44702
130	142.9	2.26715	2.31333	2.54898
155	145.1	2.42163	2.47700	2.72153

4.1.2.2 Model comparison for electric machines

Table 4.14 compares the results of fitting different models to the data. The model with the lowest value of the AIC is model M, which has been used to generate the forecasts. Table 4.15 summarises the performance of the currently selected model M in fitting the historical data.

(A) Random walk

(B) Constant mean = 110.608

(C) Linear trend = 100.996 + 0.122447 t

(M) ARIMA(2,0,0)x$(2,1,2)_{12}$ with constant

(N) ARIMA(2,0,0)x$(2,1,2)_{12}$

(O) ARIMA(1,1,0)x$(2,1,2)_{12}$

(P) ARIMA(1,1,0)x$(2,1,2)_{12}$ with constant

(Q) ARIMA(0,1,1)x$(0,1,2)_{12}$

Table 4.14 Model comparison of electric machines

Model	RMSE	MAE	MAPE	ME	MPE	AIC
(A)	7.82156	5.98037	5.42775	-0.00385338	-0.242577	4.24820
(B)	10.45790	8.08041	7.26306	0.00576924	-0.792385	4.84855
(C)	8.66202	6.88130	6.31290	-0.00316562	-0.561769	4.48456
(M)	5.85547	4.48266	4.05884	-0.16994900	-0.390084	3.62450
(N)	5.95797	4.46488	4.01744	0.27000200	-0.006919	3.64638
(O)	6.09703	4.63977	4.20808	0.08433140	-0.110372	3.67971
(P)	6.10219	4.64552	4.21699	0.03210860	-0.142934	3.69422
(Q)	6.26228	4.84591	4.38771	-0.06053910	-0.335397	3.70755

Table 4.15 summarises the results of 5 tests run on the residuals to determine whether each model is adequate for the data. An “OK” means that the model passes the test. The current selected model M passed all the 5 tests. Since no tests are statistically significant at the 95% or higher confidence level, the current model is adequate for the data.

Table 4.15 Estimation period of electric machines

Model	RMSE	RUNS	RUNM	AUTO	MEAN	VAR
(A)	7.82156	*	***	***	OK	OK
(B)	10.45790	OK	***	***	***	***
(C)	8.66202	OK	***	***	OK	**
(M)	5.85547	OK	OK	OK	OK	OK
(N)	5.95797	OK	OK	*	OK	OK
(O)	6.09703	OK	OK	**	OK	OK
(P)	6.10219	OK	OK	**	OK	OK
(Q)	6.26228	OK	OK	OK	OK	OK

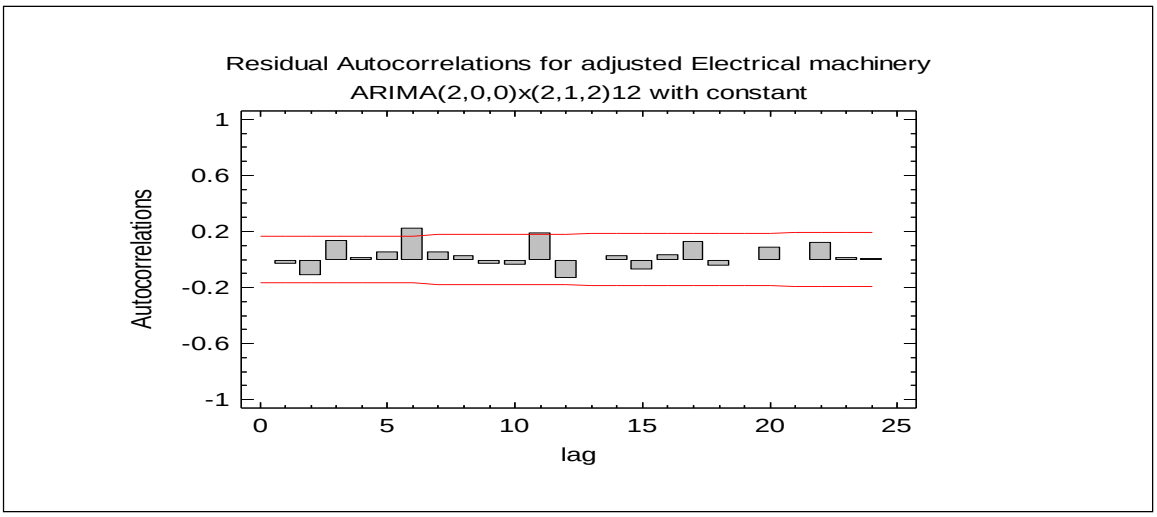


Figure 4.12 Residuals autocorrelation for adjusted electric machines

This ARIMA model selected will be used in forecasting future values of electric machines. The data cover 156 time periods. Currently, ARIMA model has been selected. This model assumes that the best forecast for future data is given by a parametric model relating the most recent data value to previous data values and previous noise.

Each value of electric machines has been adjusted before the model was fitted. Table 4.16 summarises the statistical significance of the terms in the forecasting model. Parameters with p-values less than 0.05 are statistically significantly different from zero at the 95% confidence level. The p-values for the parameters AR(2), SAR(2) and SMA(2) are less than 0.05, so it is significantly different from zero. The estimated standard deviation of the input white noise is 6.27869.

Table 4.16 Estimates of parameters of electric machines

Parameter	Estimate	Stnd. Error	T	P-value
AR(1)	0.355515	0.0821945	4.32529	0.000029
AR(2)	0.327082	0.0821918	3.97950	0.000112
SAR(1)	1.128970	0.0741390	15.22780	0.000000
SAR(2)	-0.542350	0.0601540	-9.01602	0.000000
SMA(1)	1.722490	0.0522924	32.93960	0.000000
SMA(2)	-0.758869	0.0495383	-15.31880	0.000000
Mean	1.662870	0.5097050	3.26241	0.001395
Constant	0.218180			

4.1.2.3 Diagnostic check for the residuals of electric machines

It is observed from the sample ACF and PACF plots of the residuals in figure 4.13 and figure 4.14 that there are some small significant values as indicated by the modified Box-Pierce (Ljung-Box) test in table 4.17, but most of the autocorrelation is modeled out. The ACF shows that there is a little correlation at lag 11 while with PACF there is a little correlation at lag 25.

Table 4.17 Modified Box Pierce Chi-Square statistic of electric machines

Modified Box-Pierce (Ljung-Box) Chi-Square statistic				
Lags	12	24	36	48
Chi-Square	19.700	29.200	47.50	56.20
DF	5	17	29	41
P-Value	0.001	0.033	0.02	0.06

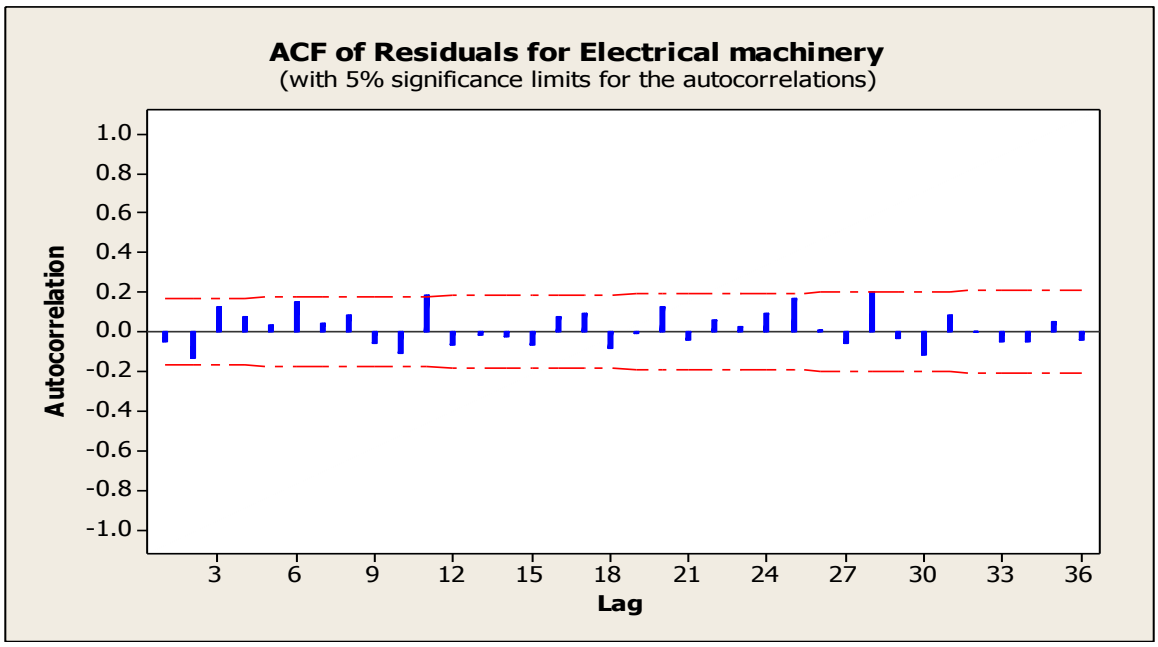


Figure 4.13 ACF of Residuals for electric machines

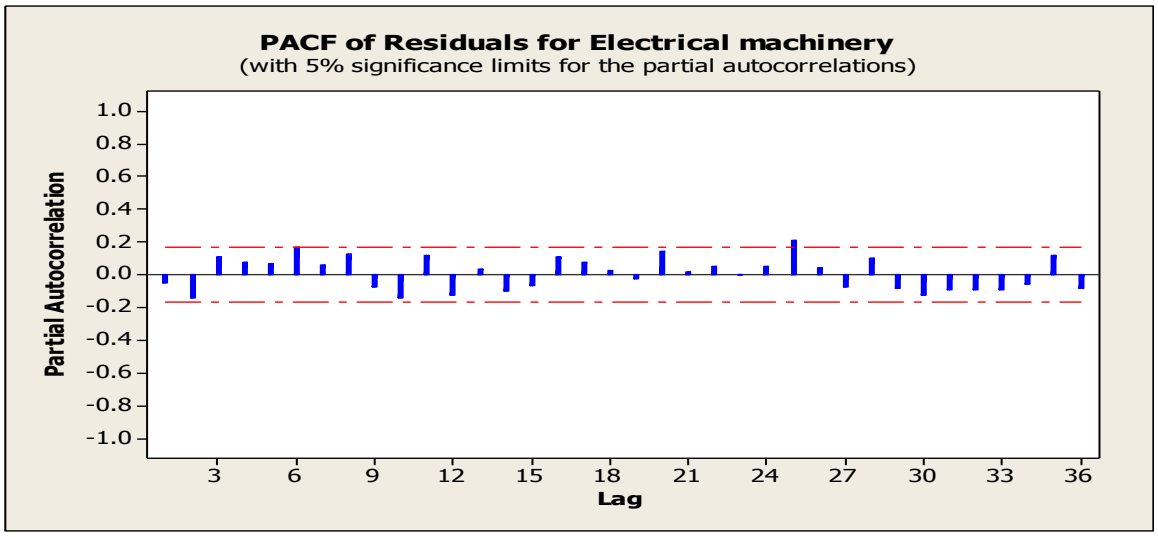


Figure 4.14 PACF of Residuals for electric machines

The 4-in-1 residual plots for the differenced data of electric machines also indicate that the fit is acceptable as indicated by figure 4.15 because it is noticed that on the normality probability plot against the percentage that the points fall approximately on a straight line and which indicate that the normality assumption holds. From the histogram of the residuals and the fitted values they assume symmetric and no patterns is formed.

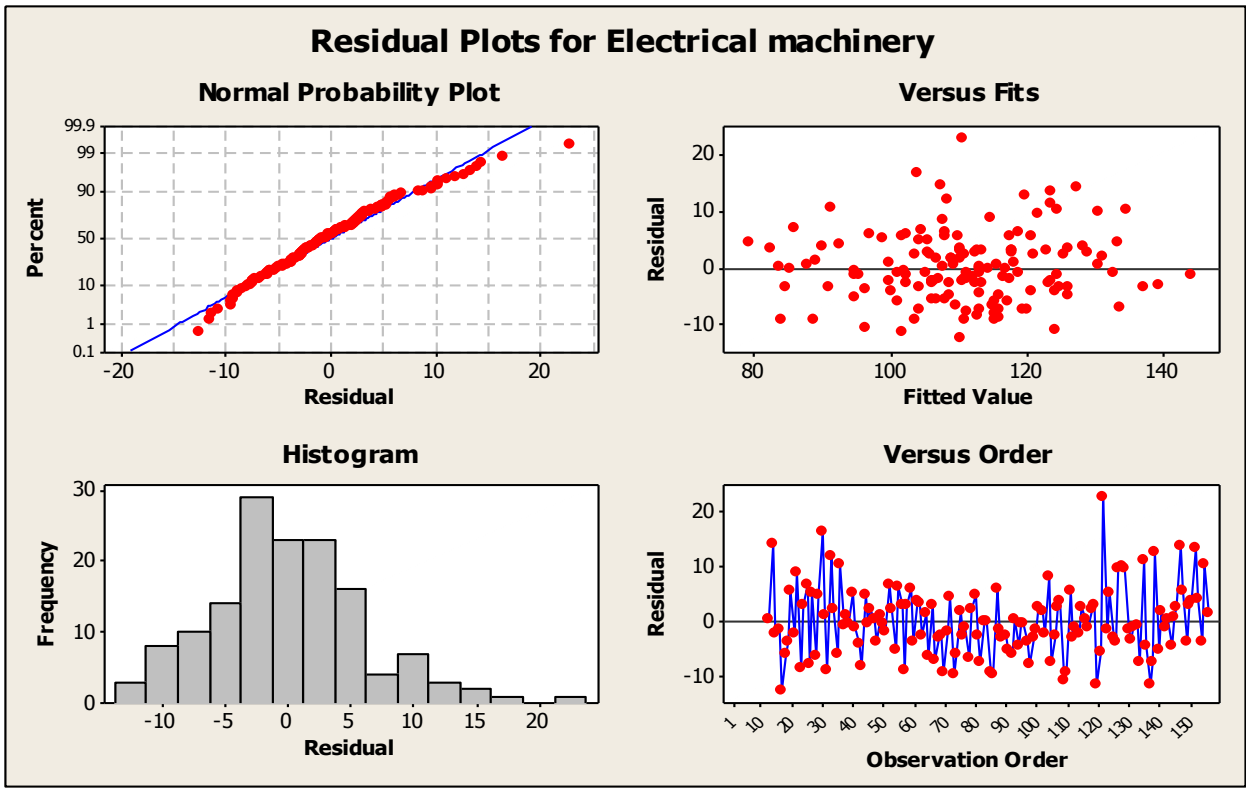


Figure 4.15 Residual Plots for Electric machines

Table 4.18 presents the forecasted values and the confidence interval of electric machines for the year 2011 using ARIMA $(2,0,0)x(2,1,2)_{12}$.

Table 4.18 Forecasts for electric machines

LL	FORECAST FOR ELECTRIC MACHINES (2011)	UL
95.7668	108.1551	120.5434
109.8226	123.0454	136.2682
119.6170	134.0101	148.4031
106.8498	121.6860	136.5221
109.6828	124.8518	140.0208
112.8557	128.2026	143.5494
109.6962	125.1576	140.6191
114.3993	129.9283	145.4572
112.1497	127.7204	143.2912
114.2017	129.7977	145.3937
120.6312	136.2427	151.8543
89.6882	105.3092	120.9302

4.1.3 Other electrical equipment

Figure 4.16 is a trend analysis for differenced data which confirms that the data is stationary at a constant variance and mean zero.

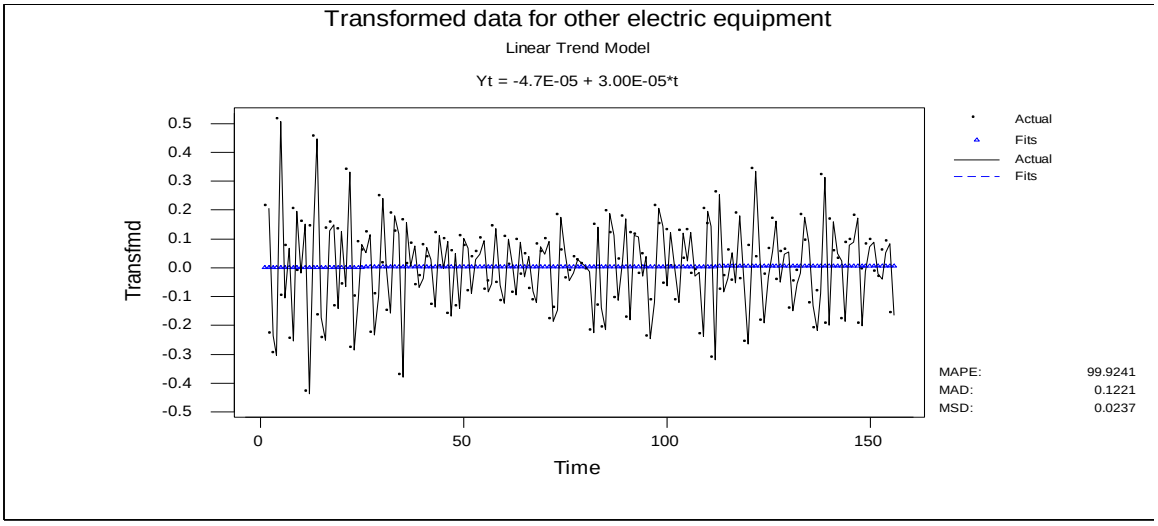
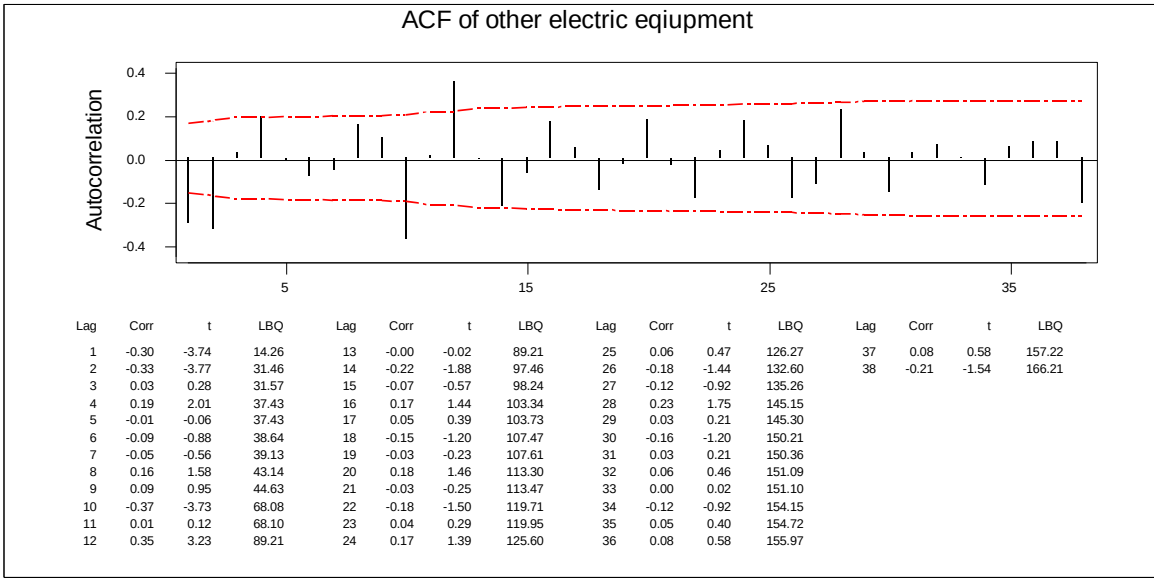


Figure 4.16 Time series plot of transformed data for other electrical equipment

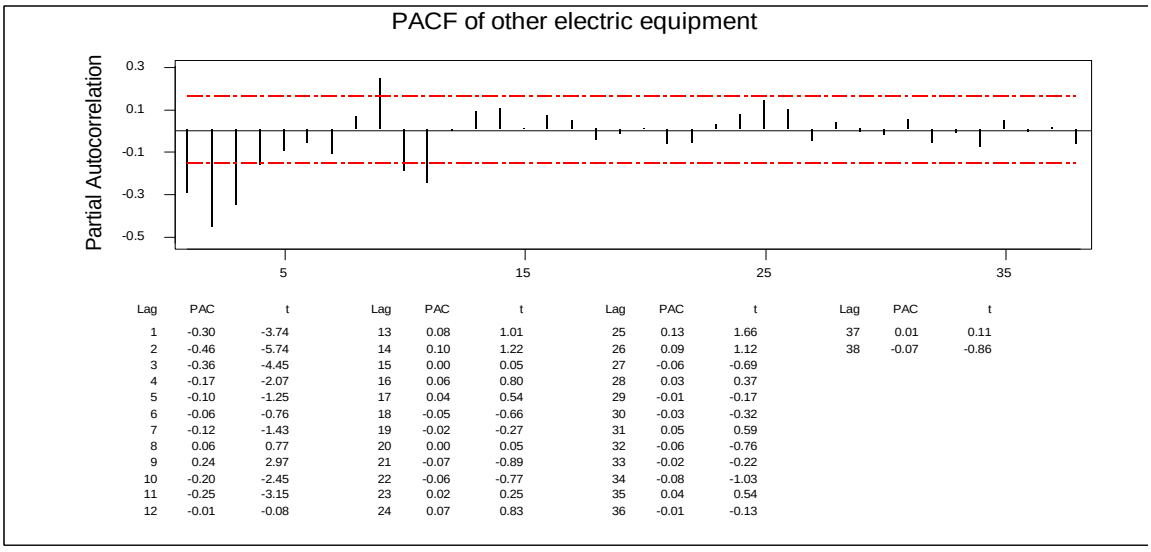
Transfmd=Transformed data

Figure 4.17 and figure 4.18 respectively show ACF and PACF for transformed data of other electrical equipment with significant values at certain lags.



Lag	Corr	t	LBQ	Lag	Corr	t	LBQ	Lag	Corr	t	LBQ	Lag	Corr	t	LBQ
1	-0.30	-3.74	14.26	13	-0.00	-0.02	89.21	25	0.06	0.47	126.27	37	0.08	0.58	157.22
2	-0.33	-3.77	31.46	14	-0.22	-1.88	97.46	26	-0.18	-1.44	132.60	38	-0.21	-1.54	166.21
3	0.03	0.28	31.57	15	-0.07	-0.57	98.24	27	-0.12	-0.92	135.26				
4	0.19	2.01	37.43	16	0.17	1.44	103.34	28	0.23	1.75	145.15				
5	-0.01	-0.06	37.43	17	0.05	0.39	103.73	29	0.03	0.21	145.30				
6	-0.09	-0.88	38.64	18	-0.15	-1.20	107.47	30	-0.16	-1.20	150.21				
7	-0.05	-0.56	39.13	19	-0.03	-0.23	107.61	31	0.03	0.21	150.36				
8	0.16	1.58	43.14	20	0.18	1.46	113.30	32	0.06	0.46	151.09				
9	0.09	0.95	44.63	21	-0.03	-0.25	113.47	33	0.00	0.02	151.10				
10	-0.37	-3.73	68.08	22	-0.18	-1.50	119.71	34	-0.12	-0.92	154.15				
11	0.01	0.12	68.10	23	0.04	0.29	119.95	35	0.05	0.40	154.72				
12	0.35	3.23	89.21	24	0.17	1.39	125.60	36	0.08	0.58	155.97				

Figure 4.17 ACF of transformed data for other electrical equipment



Lag	PAC	t	Lag	PAC	t	Lag	PAC	t	Lag	PAC	t
1	-0.30	-3.74	13	0.08	1.01	25	0.13	1.66	37	0.01	0.11
2	-0.46	-5.74	14	0.10	1.22	26	0.09	1.12	38	-0.07	-0.86
3	-0.36	-4.45	15	0.00	0.05	27	-0.06	-0.69			
4	-0.17	-2.07	16	0.06	0.80	28	0.03	0.37			
5	-0.10	-1.25	17	0.04	0.54	29	-0.01	-0.17			
6	-0.06	-0.76	18	-0.05	-0.66	30	-0.03	-0.32			
7	-0.12	-1.43	19	-0.02	-0.27	31	0.05	0.59			
8	0.06	0.77	20	0.00	0.05	32	-0.06	-0.76			
9	0.24	2.97	21	-0.07	-0.89	33	-0.02	-0.22			
10	-0.20	-2.45	22	-0.06	-0.77	34	-0.08	-1.03			
11	-0.25	-3.15	23	0.02	0.25	35	0.04	0.54			
12	-0.01	-0.08	24	0.07	0.83	36	-0.01	-0.13			

Figure 4.18 PACF of transformed data for other electrical equipment

4.1.3.1 Outlier identification: Other electrical equipment using Grubbs' test

Test statistic = 2.48462

P-Value = 1.0

This analysis identifies and treats potential outliers in samples from normal populations. Table 4.19 to table 4.21 display the usual estimates of the mean and standard deviation, together with estimates which are designed to be resistant to outliers. For the 154 values of other

electrical equipment, the sample mean and sigma are 129.375 and 21.3615, respectively. The corresponding Winsorized estimates of which 15% of the largest and smallest data values are replaced by values from the interior of the sample, are 129.445 and 21.9618 respectively.

Table 4.19 Location estimates of other electrical equipment

Sample mean	129.375
Sample median	128.850
Trimmed mean	128.715
Winsorized mean	129.445

Trimming: 15%

Table 4.20 Scale estimates of other electrical equipment

Sample std. deviation	21.3615
MAD/0.6745	17.3462
Sbi	21.6664
Winsorized sigma	21.9618

Table 4.21 shows the smallest and largest values of other electrical equipment. The Studentised values measure how many standard deviations each value is from the sample mean of 129.375. Since the p-value for Grubbs' test is greater or equal to 0.05, that value is not a significant outlier at the 5% significance level, assuming that all the other values follow a normal distribution. Similar scores are displayed after deleting each point one at a time when computing the sample statistics, and when the mean and standard deviation are based on the MAD. Values of the modified scores greater than 3.5 in absolute value are declared as outliers, of which there is none, as shown in table 4.22 and figure 4.19.

Table 4.21 The 95% confidence intervals for the mean of other electrical equipment

	Lower Limit	Upper Limit
Standard	125.975	132.776
Winsorized	125.256	133.635

Table 4.22 Sorted Values of other electrical equipment

Row	Value	Studentised Values Without Deletion	Studentised Values With Deletion	Modified MAD Z-Score
85	76.3	-2.48462	-2.54489	-3.02949
97	80.7	-2.27864	-2.32610	-2.77583
4	81.3	-2.25056	-2.29644	-2.74124
12	90.2	-1.83392	-1.86056	-2.22816
96	91.0	-1.79647	-1.82173	-2.18204
152	174.8	2.12647	2.16582	2.64900
154	176.9	2.22478	2.26924	2.77006
22	178.0	2.27627	2.32360	2.83348
151	178.4	2.29500	2.34339	2.85654
135	178.8	2.31372	2.36321	2.87960

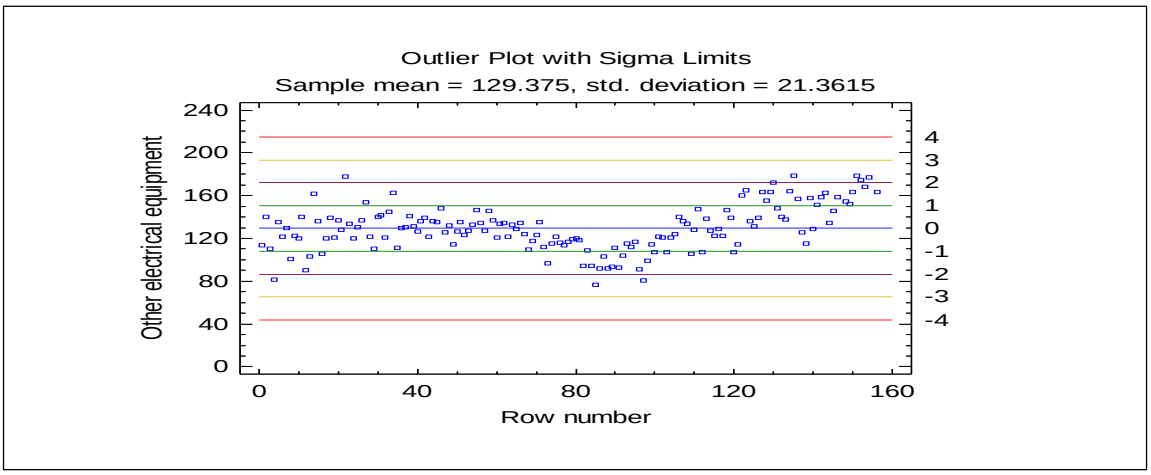


Figure 4.19 Outlier plot of other electrical equipment

4.1.3.2 Model comparison for other electrical equipment

Table 4.23 compares the results of fitting different models to the data. The model with the lowest value of the AIC is model M, which has been used to generate the forecasts. Table 4.23 summarises the performance of the currently selected model M in fitting the historical data.

(A) Random walk

(B) Constant mean = 130.159

(C) Linear trend = 116.72 + 0.171198 t

(M) ARIMA(0,1,1)x$(0,1,2)_{12}$

(N) ARIMA(0,1,1)x$(1,1,1)_{12}$

(O) ARIMA(1,0,1)x$(0,1,2)_{12}$

(P) ARIMA(0,1,1)x$(0,1,2)_{12}$ with constant

(Q) ARIMA(0,1,1)x$(0,1,1)_{12}$

Table 4.23 Model comparison of other electrical equipment

Model	RMSE	MAE	MAPE	ME	MPE	AIC
(A)	16.9550	12.75770	10.09480	-0.021849	-0.87531	5.79546
(B)	21.4807	16.30910	13.03420	-0.001665	-2.61263	6.28815
(C)	19.9712	15.73170	12.77300	-0.008486	-2.31434	6.15525
(M)	13.2358	10.05160	7.90262	-0.261074	-1.08007	5.20431
(N)	13.2481	10.01590	7.87201	-0.470550	-1.21133	5.20617
(O)	13.2029	9.88524	7.69024	0.865738	-0.13887	5.21215
(P)	13.2129	9.85558	7.79181	-0.769235	-1.46267	5.21367
(Q)	13.3902	10.33220	8.10228	0.106072	-0.84083	5.21469

Table 4.24 summarises the results of five tests run on the residuals to determine whether each model is adequate for the data. An “OK” means that the model passes the test. The currently selected model, i.e. model M passes all the five tests. Since no tests are statistically significant at the 95% or higher confidence level, the current model is adequate for the data.

Table 4.24 Estimation period of other electrical equipment

Model	RMSE	RUNS	RUNM	AUTO	MEAN	VAR
(A)	16.9550	OK	**	***	OK	*
(B)	21.4807	OK	***	***	OK	***
(C)	19.9712	OK	***	***	**	***
(M)	13.2358	OK	OK	OK	OK	OK
(N)	13.2481	OK	OK	OK	OK	OK
(O)	13.2029	OK	OK	OK	OK	OK
(P)	13.2129	OK	OK	OK	OK	OK
(Q)	13.3902	OK	OK	OK	OK	OK

Figure 4.20 corfirms that the selected model, i.e. ARIMA(0,1,1)x(0,1,2)$_{12}$ of other electrical equipmenrt is adequate since there is no autocorrelation in the residuals.

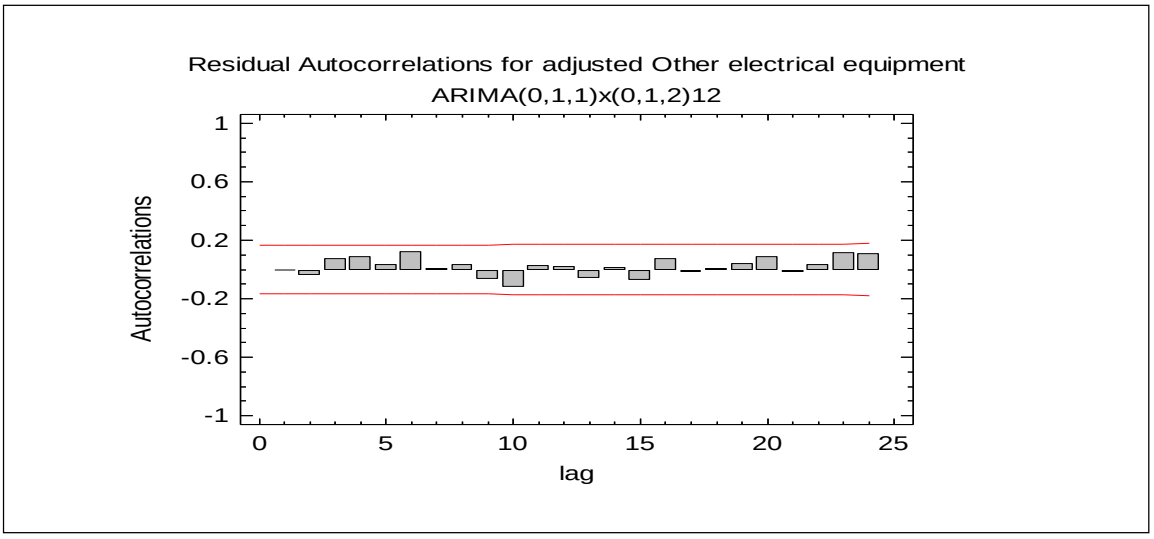


Figure 4.20 Residuals autocorelation for adjusted other electrical equipment

Table 4.25 summarises the statistical significance of the terms in the forecasting model. Terms with p-values less than 0.05 are statistically significantly different from zero at the 95% confidence level.

The p-values for the parameters are less than 0.05, and hence significantly different from zero. The estimated standard deviation of the input white noise is 6.27869.

Table 4.25 Estimates of parameters of other electrical equipment

Parameter	Estimate	Stnd. Error	T	P-value
AR(1)	0.355515	0.0821945	4.32529	0.000029
AR(2)	0.327082	0.0821918	3.97950	0.000112
SAR(1)	1.128970	0.0741390	15.22780	0.000000
SAR(2)	-0.542350	0.0601540	-9.01602	0.000000
SMA(1)	1.722490	0.0522924	32.93960	0.000000
SMA(2)	-0.758869	0.0495383	-15.31880	0.000000
Mean	1.662870	0.5097050	3.26241	0.001395
Constant	0.218180			

Estimated white noise variance = 39.4219 with 137 degrees of freedom
Estimated white noise standard deviation = 6.27869

4.1.3.3 Diagnostic test for residuals

The modified Box-Pierce test suggests that there is no autocorrelation left in the residuals (Table 4.26). This is confirmed by the ACF and PACF plots of the residuals given in figure 4.21 and figure 4.22 respectively.

Table 4.26 Modified Box-Pierce Chi-Square statistic of other electrical equipment

Modified Box-Pierce (Ljung-Box) Chi-Square statistic				
Lags	12	24	36	48
Chi-Square	9.400	16.800	30.400	47.400
DF	8	20	32	44
P-Value	0.309	0.666	0.545	0.336

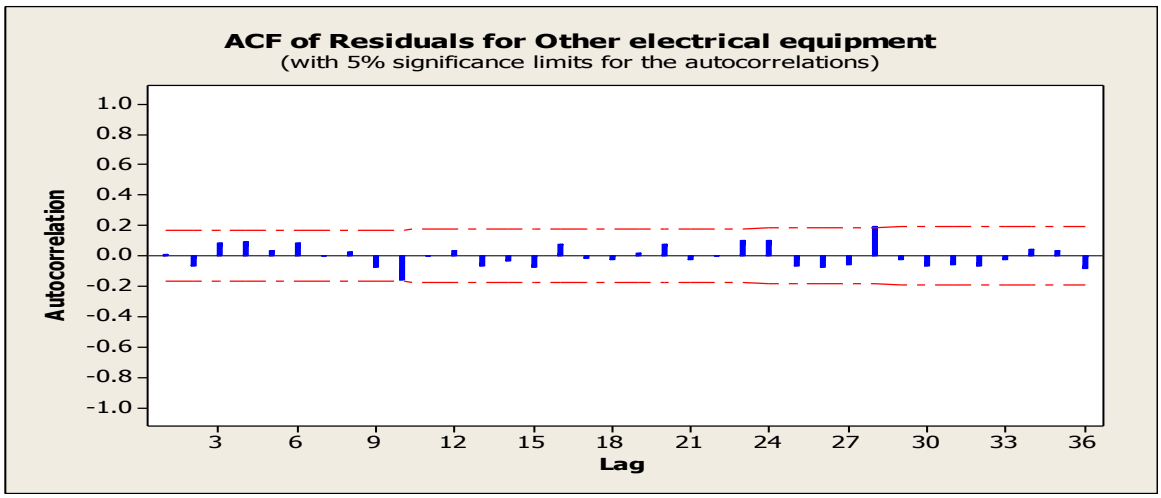


Figure 4.21 ACF of Residuals for other electrical equipment

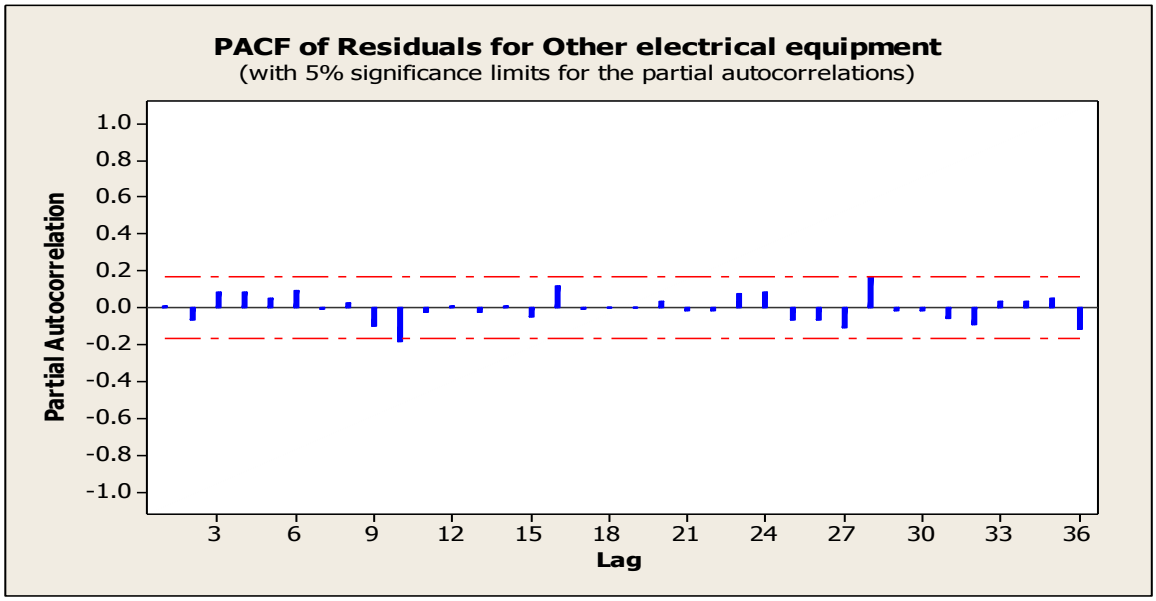


Figure 4.22 PACF of Residuals for other electrical equipment

The last diagnostic check is the 4-in-1 residual plots in figure 4.23 which is the normal probability plot, residual versus the fitted value, histogram of the residuals and time series plot of the residuals.

Figure 4.23 indicates that the fit is acceptable because from the normality plot it can be observed that the residuals are independent and normally distributed since they fall approximately on a straight line. The histogram plot shows that the residuals are normally distributed since the plot is approximately symmetric about the mean zero. The fitted values against the residuals confirm that the model is adequate since there is no correlation. The time series of the residuals also indicate that correlation does not exist.

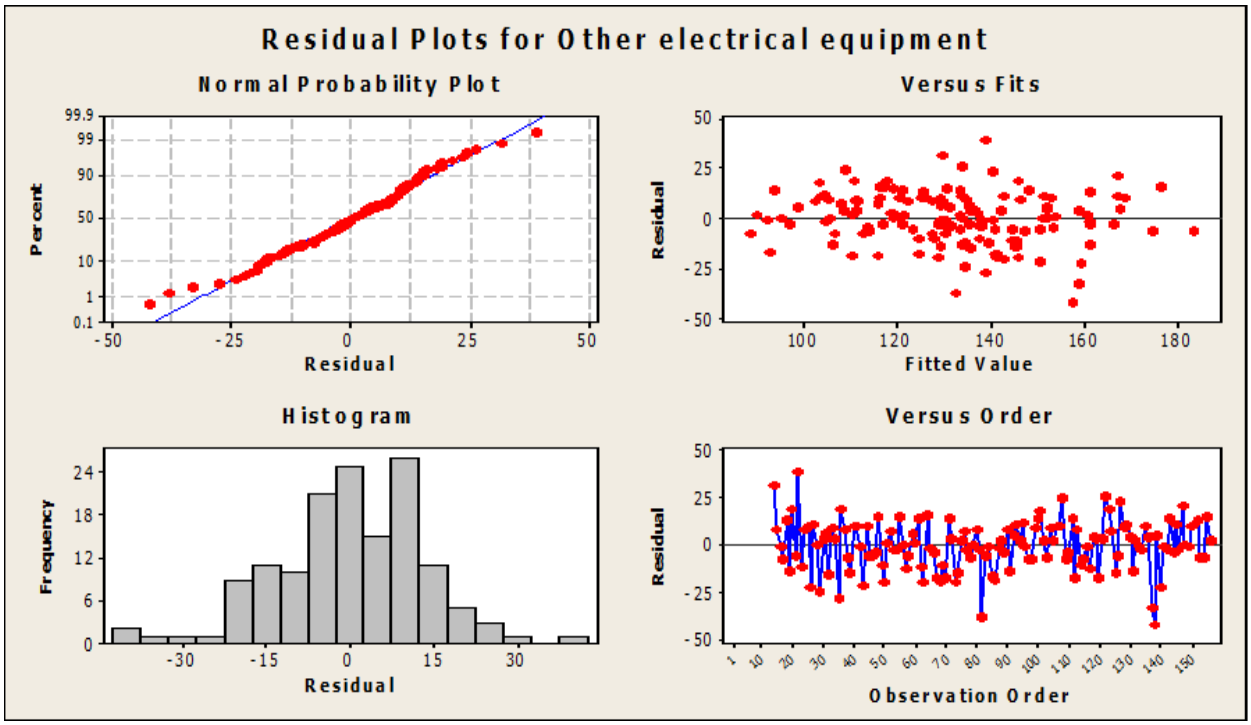


Figure 4.23 Residual Plots for other electrical equipment

Table 4.27 presents the forecasted values and the confidence interval for the year 2011 of other electrical equipment using ARIMA (0, 1, 1)x(0, 1, 2)$_{12}$.

Table 4.27 Forecasts for other electrical equipment

LL	FORECASTS FOR OTHER ELECTRICAL APPLIANCES (Jan-Dec 2011)	UL
136.7026	162.9260990	189.1496
157.2820	184.3282224	211.3744
168.1752	196.0198406	223.8645
144.5108	173.1315525	201.7523
147.7548	177.1312691	206.5077
153.8972	184.0103967	214.1235
160.1591	190.9913399	221.8236
156.2398	187.7747437	219.3097
158.2721	190.4944466	222.7168
168.0046	200.8999792	233.7954
165.8346	199.3895641	232.9445
145.0328	179.2345470	213.4363

4.1.4 Communication apparatus

Figure 4.24 is a trend analysis for differenced communication apparatus data which confirms that the data is stationary at a constant variance and mean zero.

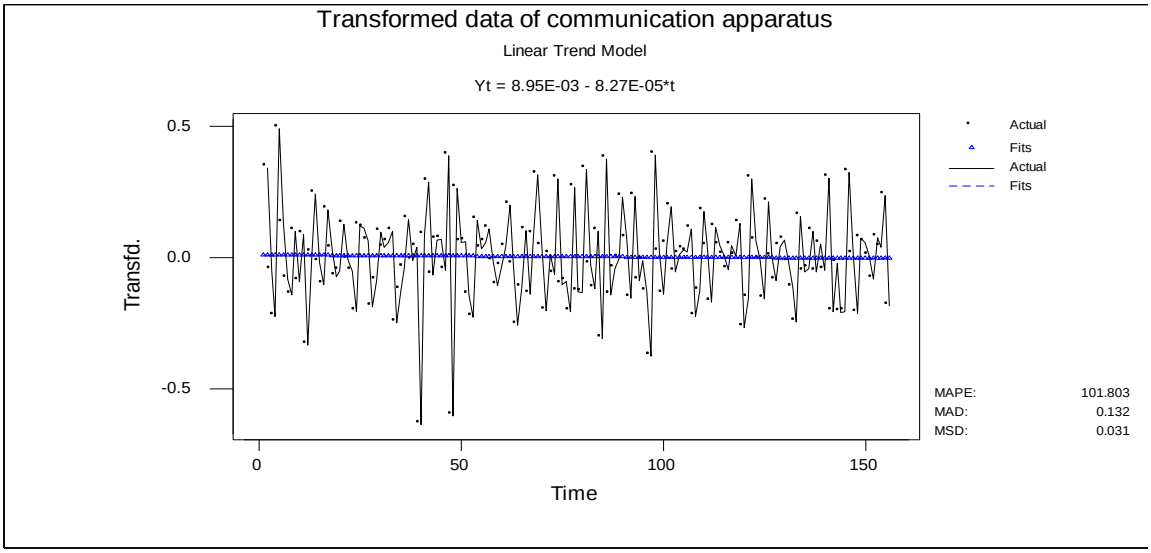
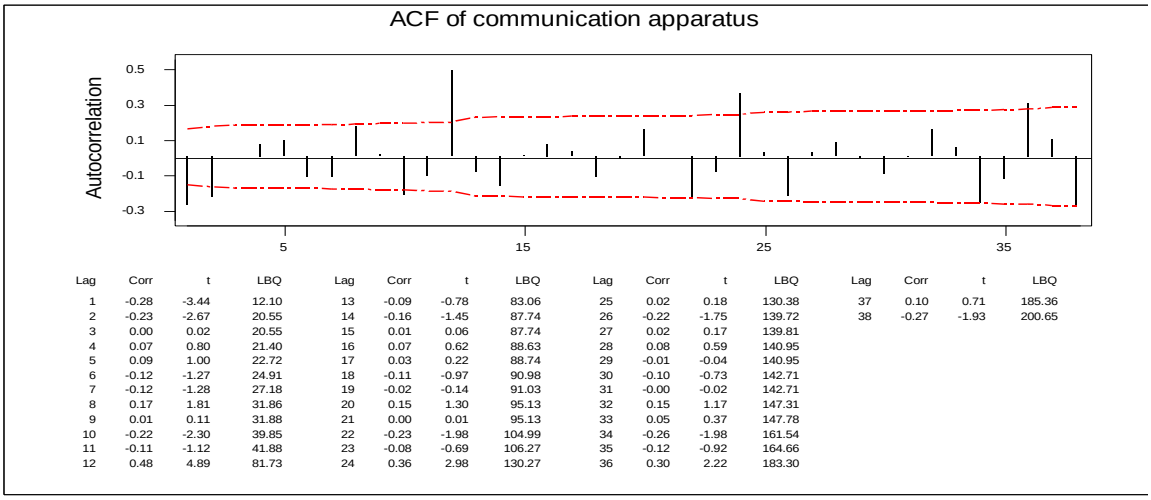


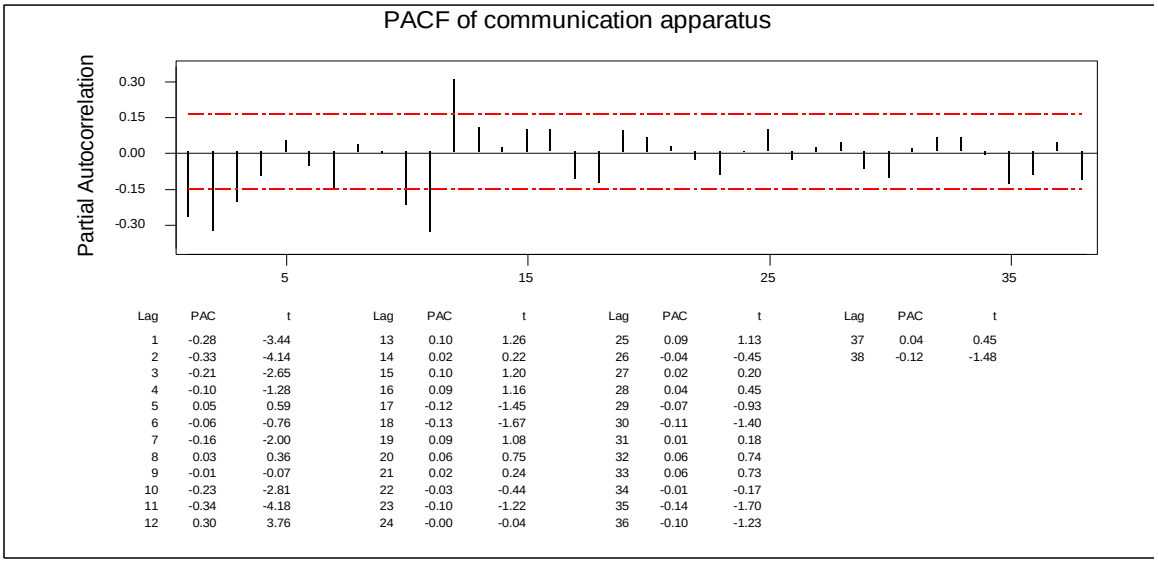
Figure 4.24 Trend of transformed data for communication apparatus
Transfd.=Transformed data

Figure 4.25 and figure 4.26 of the ACF and PACF respectively show that there are large positive correlations at certain lags of the ACF plot and a negative correlation at certain lags of the PACF plot.



Lag	Corr	t	LBQ	Lag	Corr	t	LBQ	Lag	Corr	t	LBQ	Lag	Corr	t	LBQ
1	-0.28	-3.44	12.10	13	-0.09	-0.78	83.06	25	0.02	0.18	130.38	37	0.10	0.71	185.36
2	-0.23	-2.67	20.55	14	-0.16	-1.45	87.74	26	-0.22	-1.75	139.72	38	-0.27	-1.93	200.65
3	0.00	0.02	20.55	15	0.01	0.06	87.74	27	0.02	0.17	139.81				
4	0.07	0.80	21.40	16	0.07	0.62	88.63	28	0.08	0.59	140.95				
5	0.09	1.00	22.72	17	0.03	0.22	88.74	29	-0.01	-0.04	140.95				
6	-0.12	-1.27	24.91	18	-0.11	-0.97	90.98	30	-0.10	-0.73	142.71				
7	-0.12	-1.28	27.18	19	-0.02	-0.14	91.03	31	-0.00	-0.02	142.71				
8	0.17	1.81	31.86	20	0.15	1.30	95.13	32	0.15	1.17	147.31				
9	0.01	0.11	31.88	21	0.00	0.01	95.13	33	0.05	0.37	147.78				
10	-0.22	-2.30	39.85	22	-0.23	-1.98	104.99	34	-0.26	-1.98	161.54				
11	-0.11	-1.12	41.88	23	-0.08	-0.69	106.27	35	-0.12	-0.92	164.66				
12	0.48	4.89	81.73	24	0.36	2.98	130.27	36	0.30	2.22	183.30				

Figure 4.25 ACF of transformed data for communication apparatus
Communication app=communication apparatus



Lag	PAC	t	Lag	PAC	t	Lag	PAC	t	Lag	PAC	t
1	-0.28	-3.44	13	0.10	1.26	25	0.09	1.13	37	0.04	0.45
2	-0.33	-4.14	14	0.02	0.22	26	-0.04	-0.45	38	-0.12	-1.48
3	-0.21	-2.65	15	0.10	1.20	27	0.02	0.20			
4	-0.10	-1.28	16	0.09	1.16	28	0.04	0.45			
5	0.05	0.59	17	-0.12	-1.45	29	-0.07	-0.93			
6	-0.06	-0.76	18	-0.13	-1.67	30	-0.11	-1.40			
7	-0.16	-2.00	19	0.09	1.08	31	0.01	0.18			
8	0.03	0.36	20	0.06	0.75	32	0.06	0.74			
9	-0.01	-0.07	21	0.02	0.24	33	0.06	0.73			
10	-0.23	-2.81	22	-0.03	-0.44	34	-0.01	-0.17			
11	-0.34	-4.18	23	-0.10	-1.22	35	-0.14	-1.70			
12	0.30	3.76	24	-0.00	-0.04	36	-0.10	-1.23			

Figure 4.26 PACF of transformed data for communication apparatus

4.1.4.1 Testing for Heteroskedasticity using ARCH test: Communication apparatus

Table 4.30 shows the Yule-Walker maximum likelihood estimates used in computing the maximum likelihood estimates. The MSE for the autoregressive model table 4.30 is 211.48 which is small compared to that of the ordinary least square (308.53) in table 4.28. The total R-squared statistic computed by autoregressive model is 0.33 (table 4.30), which is too small in helping to improve the prediction of the next communication apparatus values. Durbin-Watson statistic indicates that there is no autocorrelation left in the residuals since the value is approximately 2 from table 4.30.

Table 4.28 Ordinary Least Square estimates of communication apparatus

Ordinary Least Squares Estimates			
SSE	47513.628	DFE	154
MSE	308.53000	Root MSE	17.5650200
SBC	1344.95941	AIC	1338.85969
MAE	28.6855313	AICC	1338.93813
MAPE	15.5386644	Regress R-Square	0.0073
Durbin-Watson	0.8617000	Total R-Square	0.0073

Table 4.29 Parameter estimates of communication apparatus

Variable	DF	Estimate	Standard Error	t Value	Approx Pr> |t|
Intercept	1	96.2380	2.8262	34.05	<.0001
Time	1	0.0331	0.0312	1.06	0.2903

Table 4.30 Yule-Walker estimates of communication apparatus

Yule-Walker Estimates			
SSE	32145.4763	DFE	152
MSE	211.48340	Root MSE	14.5424
SBC	1294.49297	AIC	1282.29354
MAE	11.4888108	AICC	1282.55844
MAPE	12.36726	Regress R-Square	0.0032
Durbin-Watson	2.0051	Total R-Square	0.3284

Table 4.31 Parameter estimates of communication apparatus

Variable	DF	Estimate	Standard Error	t Value	Approx Pr> |t|
Intercept	1	95.2082	5.6301	16.91	<.0001
time	1	0.0430	0.0619	0.70	0.4880

Figure 4.27 shows the standardised residuals against the fitted values. The points in the plot seem to be fluctuating randomly around zero in an un-patterned fashion. The assumptions of zero mean and constant variance of the random errors hold.

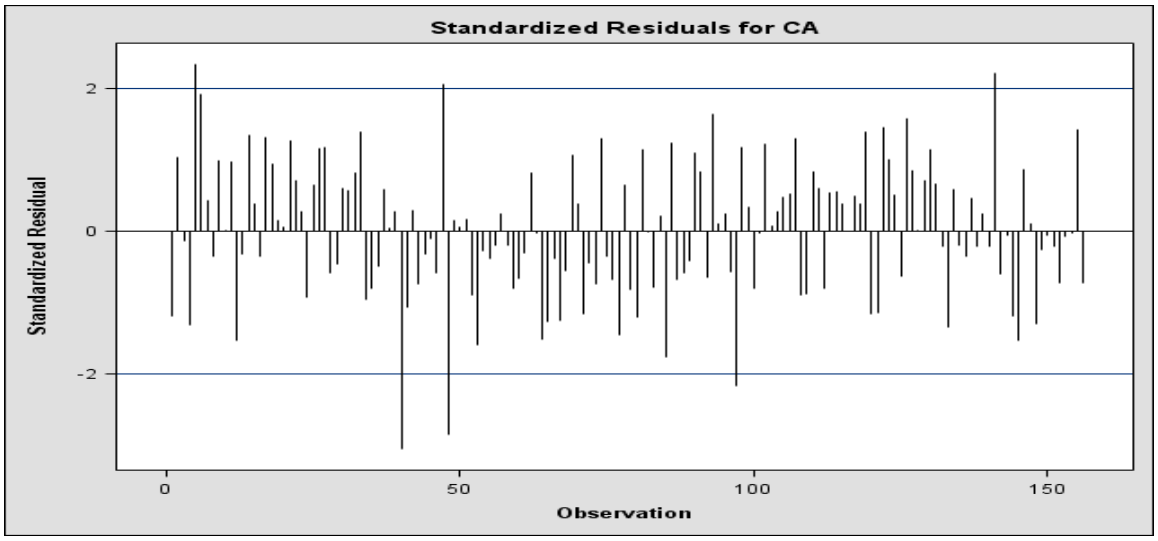


Figure 4.27 Standardised residuals for communication apparatus

Figure 4.28 depicts the predicted values and actual values of the accumulators. The predicted values seem to follow the pattern formed by the actual values of accumulators, which indicates that the model fitted is good.

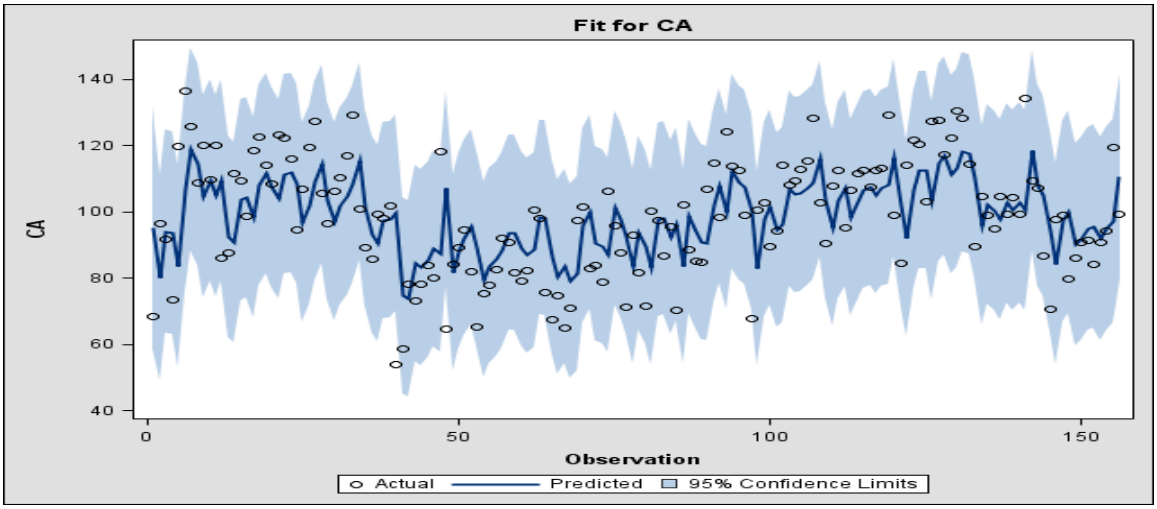


Figure 4.28 Predicted values versus actual plot for communication apparatus

Figure 4.29 and figure 4.30 show that the residuals are normally distributed since the Q-Q plot dots lie on a straight line.

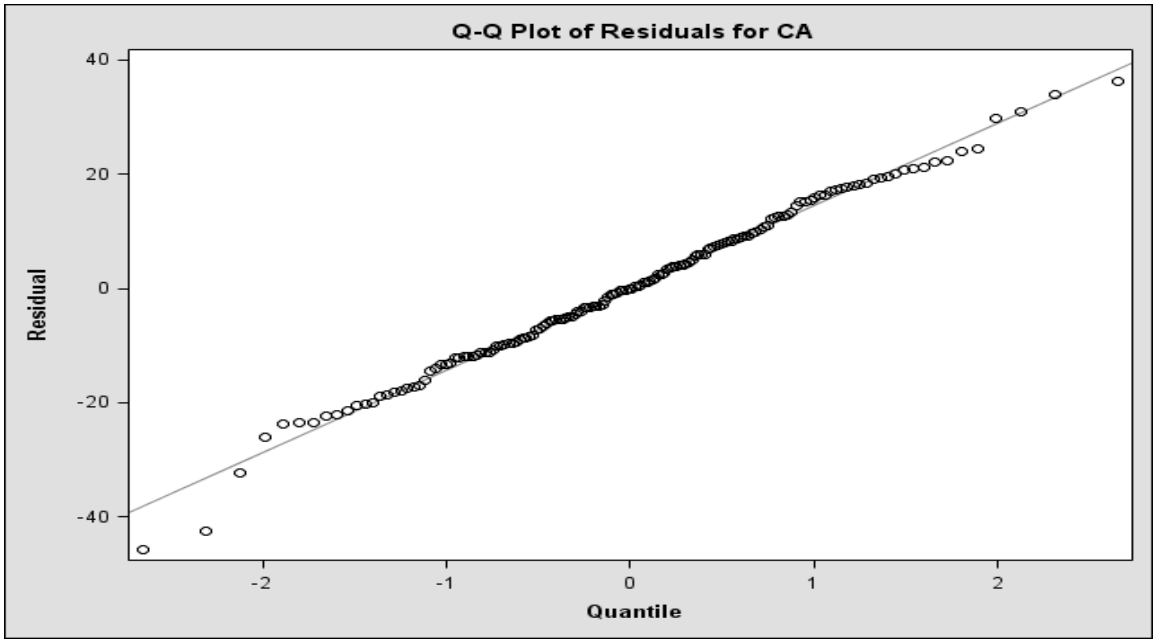


Figure 4.29 The Q-Q plot of residuals for communication apparatus

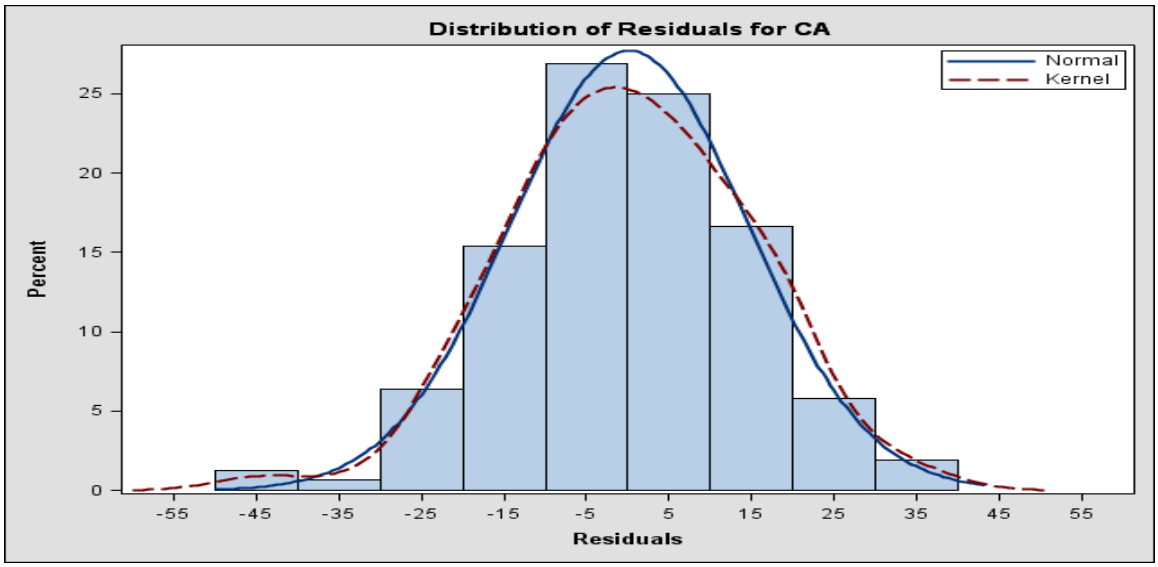


Figure 4.30 Histogram of the residuals for communication apparatus

Figure 4.31 presents the ACF of the residual plot depicting no autocorrelation, with the exception of a high autocorrelation of 0.4 at lag 12. It can be concluded that the overall model is a good model to be fitted.

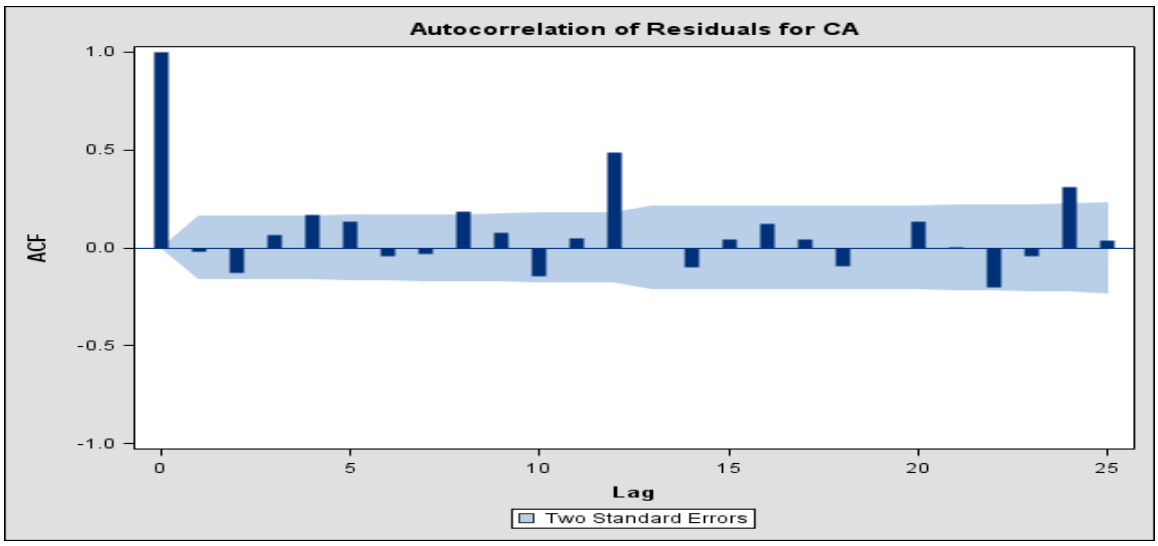


Figure 4.31 Autocorrelation of Residuals plot for communication apparatus

Figure 4.32 presents the PACF of residual plot indicating no autocorrelation left except at lag 12. It can thus be concluded that the model shows a good fit.

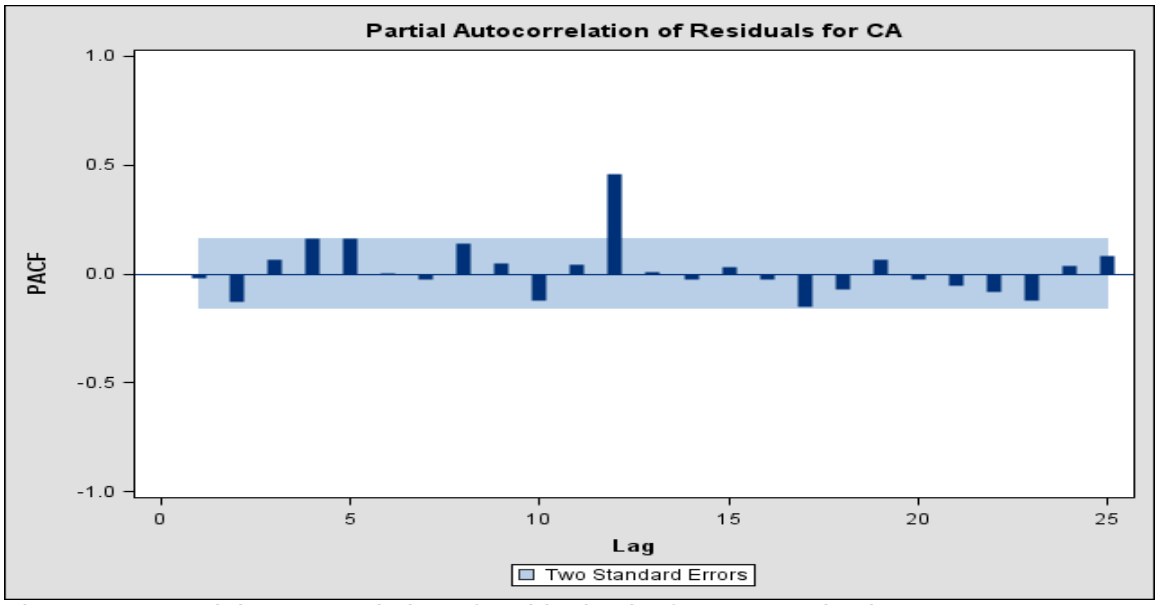


Figure 4.32 Partial autocorrelation of residuals plot for communication apparatus

4.1.5 Accumulators

Figure 4.33 is a trend analysis for differenced accumulators data which confirms that the data is stationary at a constant variance and mean zero.

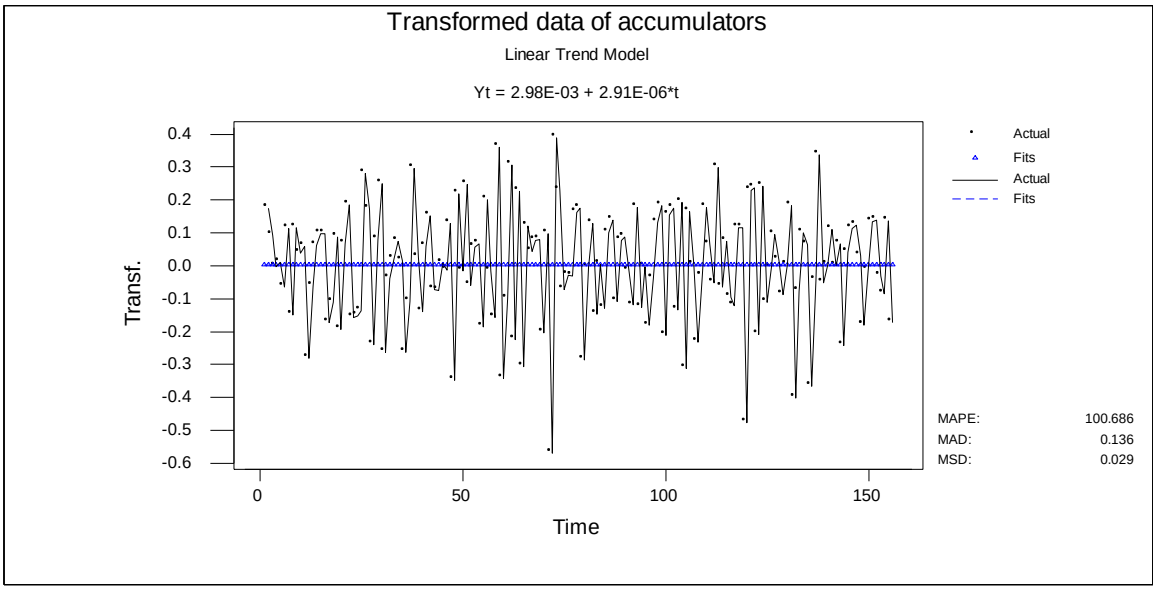
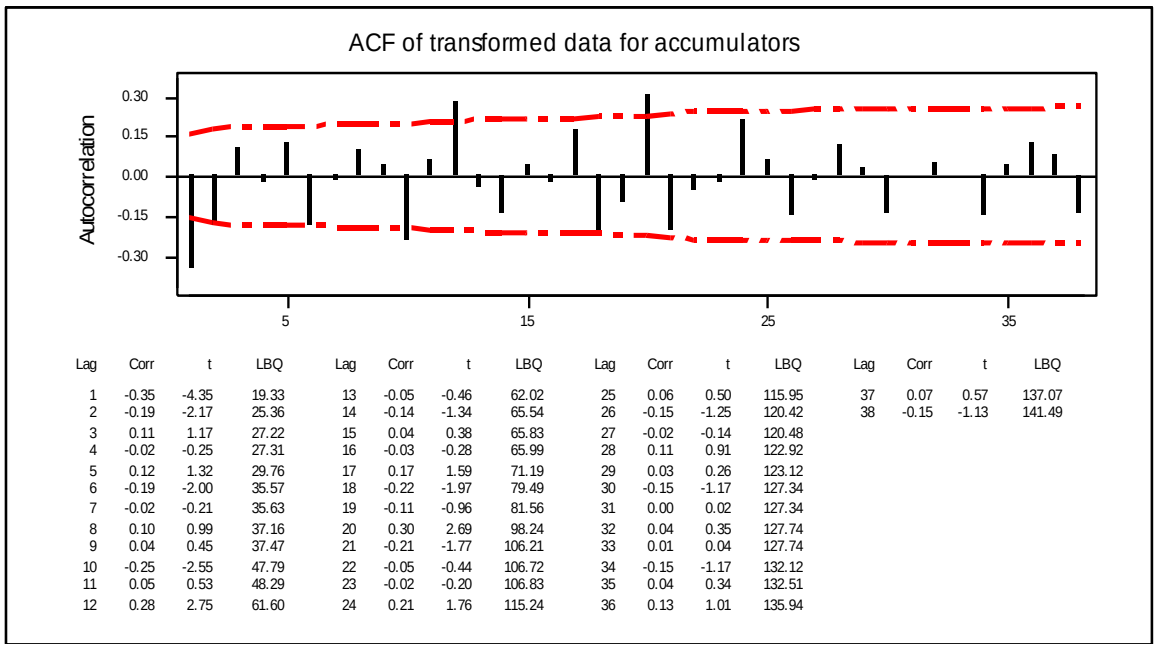


Figure 4.33 Trend analysis for differenced accumulators

Transf=transformed data

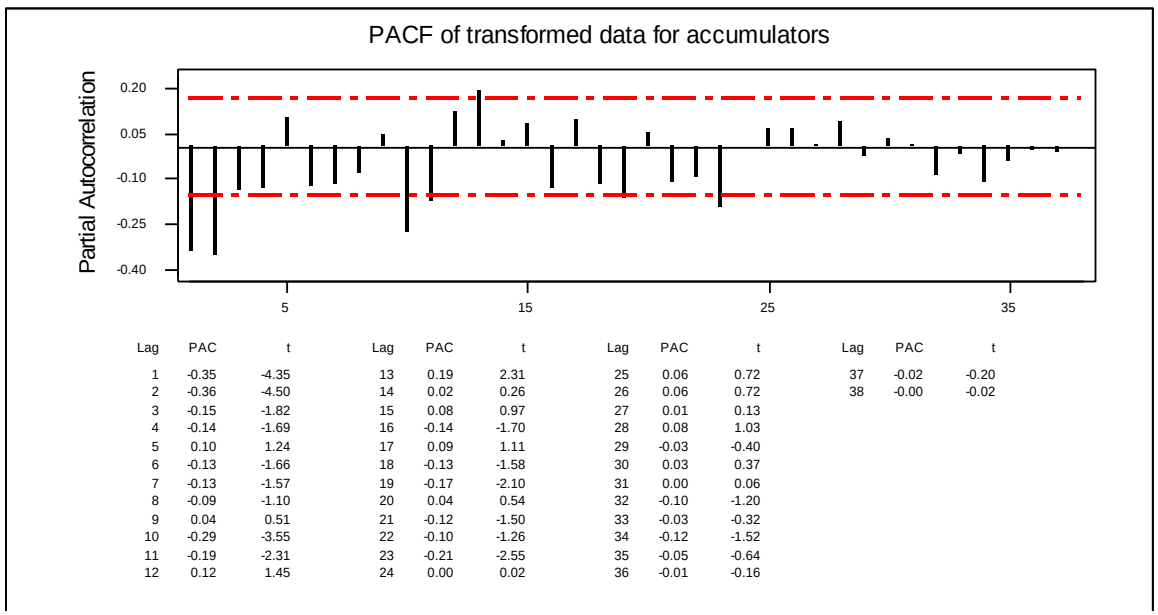
As shown from ACF plot in figure 4.34 most of the correlations are small. There are fairly large negative correlations at certain lags (lag 1 and 10) and positive correlations of 0.28 and 0.30 at lag 12 and lag 20, respectively.

As shown from ACF plot in figure 4.35 most of the correlations are small. There are fairly large negative correlations at certain lags (lag 1, 2 and 10) and a positive correlation of 0.19 at lag 13.



Lag	Corr	t	LBQ	Lag	Corr	t	LBQ	Lag	Corr	t	LBQ	Lag	Corr	t	LBQ
1	-0.35	-4.35	19.33	13	-0.05	-0.46	62.02	25	0.06	0.50	115.95	37	0.07	0.57	137.07
2	-0.19	-2.17	25.36	14	-0.14	-1.34	65.54	26	-0.15	-1.25	120.42	38	-0.15	-1.13	141.49
3	0.11	1.17	27.22	15	0.04	0.38	65.83	27	-0.02	-0.14	120.48				
4	-0.02	-0.25	27.31	16	-0.03	-0.28	65.99	28	0.11	0.91	122.92				
5	0.12	1.32	29.76	17	0.17	1.59	71.19	29	0.03	0.26	123.12				
6	-0.19	-2.00	35.57	18	-0.22	-1.97	79.49	30	-0.15	-1.17	127.34				
7	-0.02	-0.21	35.63	19	-0.11	-0.96	81.56	31	0.00	0.02	127.34				
8	0.10	0.99	37.16	20	0.30	2.69	98.24	32	0.04	0.35	127.74				
9	0.04	0.45	37.47	21	-0.21	-1.77	106.21	33	0.01	0.04	127.74				
10	-0.25	-2.55	47.79	22	-0.05	-0.44	106.72	34	-0.15	-1.17	132.12				
11	0.05	0.53	48.29	23	-0.02	-0.20	106.83	35	0.04	0.34	132.51				
12	0.28	2.75	61.60	24	0.21	1.76	115.24	36	0.13	1.01	135.94				

Figure 4.34 ACF of transformed data for accumulators



Lag	PAC	t	Lag	PAC	t	Lag	PAC	t	Lag	PAC	t
1	-0.35	-4.35	13	0.19	2.31	25	0.06	0.72	37	-0.02	-0.20
2	-0.36	-4.50	14	0.02	0.26	26	0.06	0.72	38	-0.00	-0.02
3	-0.15	-1.82	15	0.08	0.97	27	0.01	0.13			
4	-0.14	-1.69	16	-0.14	-1.70	28	0.08	1.03			
5	0.10	1.24	17	0.09	1.11	29	-0.03	-0.40			
6	-0.13	-1.66	18	-0.13	-1.58	30	0.03	0.37			
7	-0.13	-1.57	19	-0.17	-2.10	31	0.00	0.06			
8	-0.09	-1.10	20	0.04	0.54	32	-0.10	-1.20			
9	0.04	0.51	21	-0.12	-1.50	33	-0.03	-0.32			
10	-0.29	-3.55	22	-0.10	-1.26	34	-0.12	-1.52			
11	-0.19	-2.31	23	-0.21	-2.55	35	-0.05	-0.64			
12	0.12	1.45	24	0.00	0.02	36	-0.01	-0.16			

Figure 4.35 PACF of transformed data for accumulators

4.1.5.1 Testing for Heteroskedasticity using ARCH test: Accumulators

Table 4.35 shows the Yule-Walker maximum likelihood estimates used in computing the maximum likelihood estimates. The MSE for the autoregressive model is 223.92 (table 4.35)

which is small compared to the ordinary least squares of 295.01 in table 4.32. The total R-squared statistic computed by autoregressive model is 0.58 (table 4.35), which is too small in helping to improve the prediction of the next accumulator values. Table 4.35 presents Durbin-Watson value of 2.0727 which indicates that most of the autocorrelation has been removed in the ACF and PACF of the residuals. Thus the residuals are independent.

Table 4.32 Ordinary Least Squares Estimates for accumulators

Ordinary Least Squares Estimates			
SSE	45432.1951	DFE	154
MSE	295.01425	Root MSE	17.17598
SBC	1337.9713	AIC	1331.87158
MAE	27.3255705	AICC	1331.95002
MAPE	13.9964523	Regress R-Square	0.4410
Durbin-Watson	1.0333	Total R-Square	0.4410

Table 4.33 Parameter estimates for accumulators

Variable	DF	Estimate	Standard Error	t Value	Approx Pr> |t|
Intercept	1	75.8612	2.7636	27.45	<.0001
Time	1	0.3366	0.0305	11.02	<.0001

Table 4.34 Estimates of Autoregressive Parameters for accumulators

Estimates of Autoregressive Parameters			
Lag	Coefficient	Standard Error	t Value
1	-0.412082	0.080225	-5.14
2	-0.147390	0.080225	-1.84

Table 4.35 Yule-Walker estimates for accumulators

Yule-Walker Estimates			
SSE	34036.5854	DFE	152
MSE	223.92490	Root MSE	14.96412
SBC	1303.33052	AIC	1291.13110
MAE	11.8931847	AICC	1291.39600
MAPE	12.1496983	Regress R-Square	0.17860
Durbin-Watson	2.0727	Total R-Square	0.58120

Table 4.36 Parameter estimates for accumulators

Variable	DF	Estimate	Standard Error	t Value	Approx Pr> |t|
Intercept	1	75.9596	5.3284	14.26	<.0001
Time	1	0.3367	0.0586	5.75	<.0001

Figure 4.36 shows the standardised residuals against the fitted values. The points in the plot seem to be fluctuating randomly around zero in an un-patterned fashion. The assumptions of zero mean and constant variance of the random errors hold.

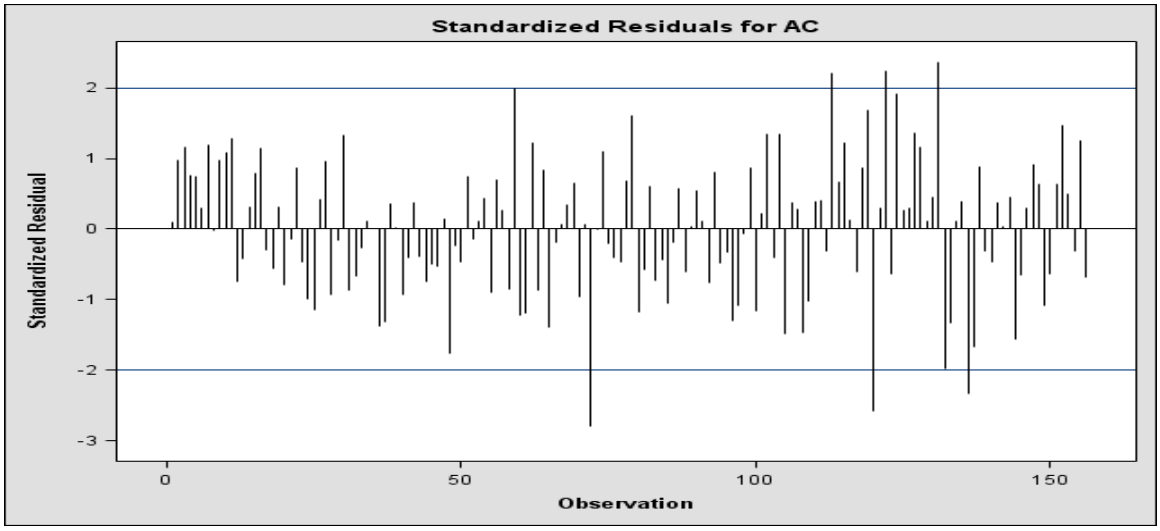


Figure 4.36 Standardised residuals plot for accumulators

Figure 4.37 indicates the predicted and actual price indices of the accumulators. The predicted values seem to follow the pattern formed by the actual values which indicates that the model is appropriately fitted.

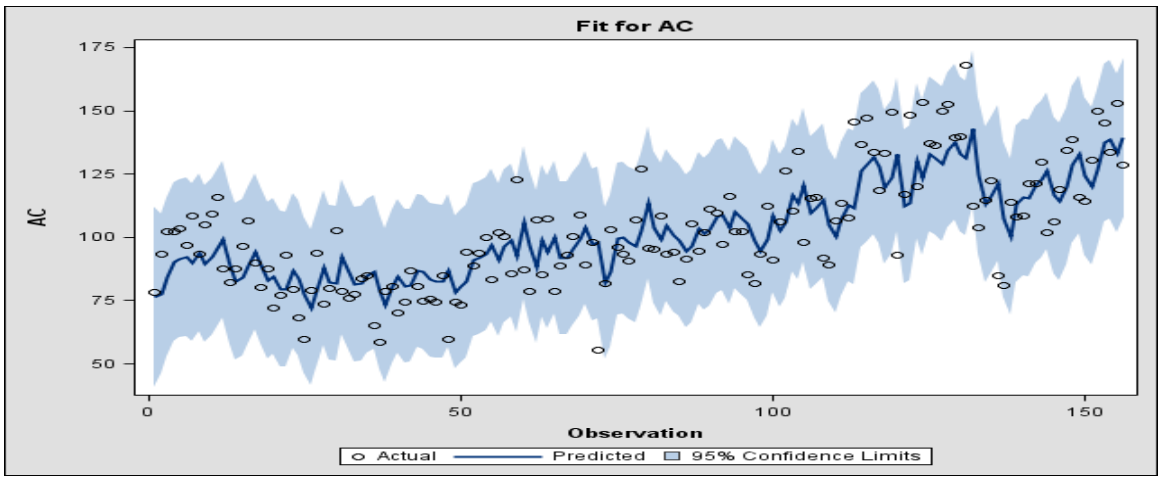


Figure 4.37 Predicted values versus actual plot for accumulators

The Q-Q plot and the histogram in figure 4.38 and figure 4.39 of the residuals respectively, assure normality of the residuals.

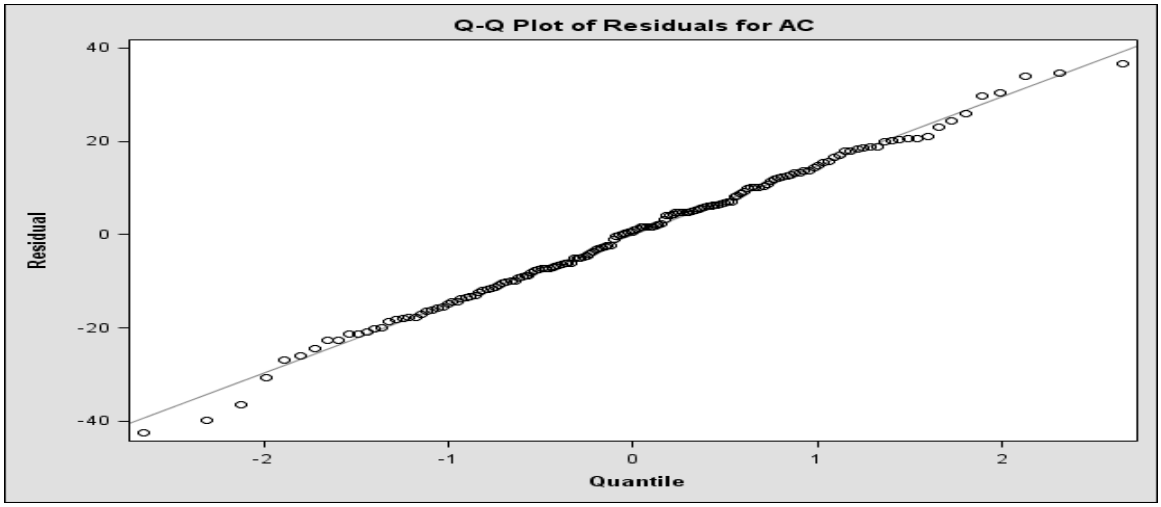


Figure 4.38 The Q-Q plot of residuals for accumulators

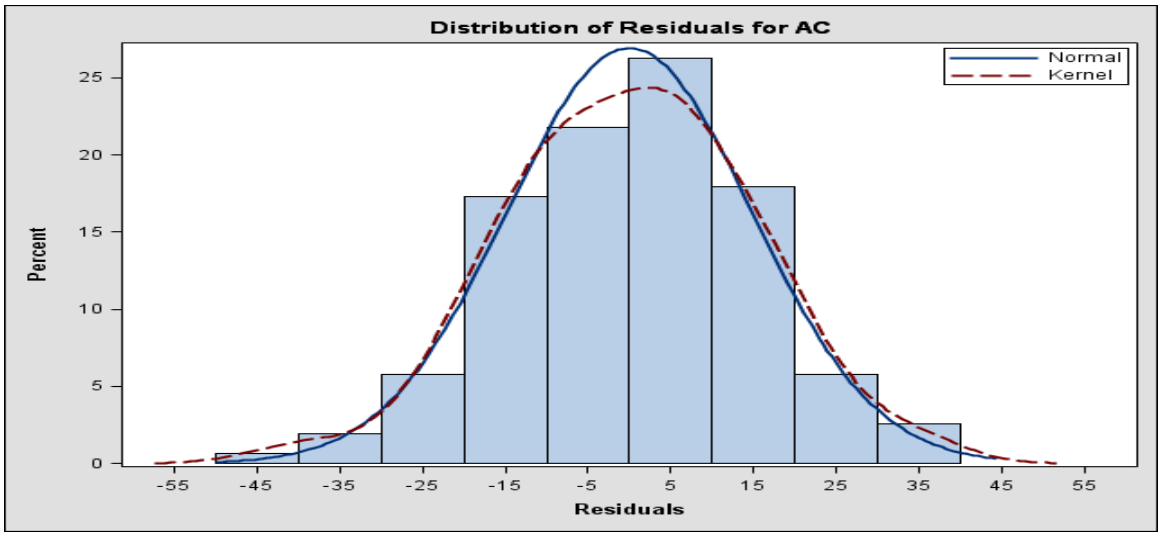


Figure 4.39 Histogram of the residuals for accumulators

The ACF of the residuals shown in figure 4.40 shows that there is still some small significant values left (at lag 12) but most of the autocorrelation has been modeled out.

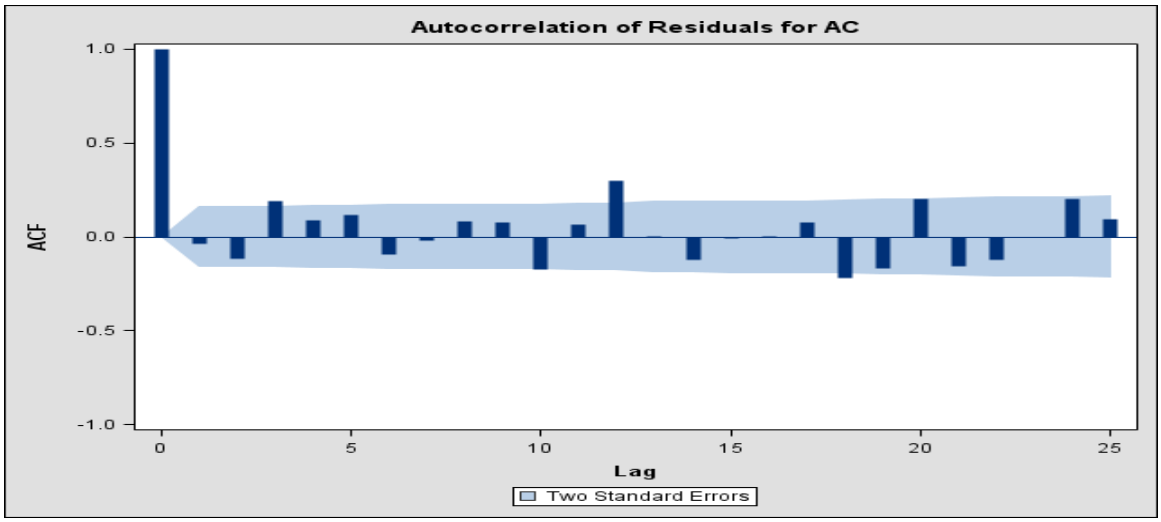


Figure 4.40 Autocorrelation of Residuals plot for accumulators

Although most of the autocorrelation is now modeled out, some small significant values are left at lag 12 as depicted from the PACF of residuals (figure 4.41). Hence the model fits quite well.

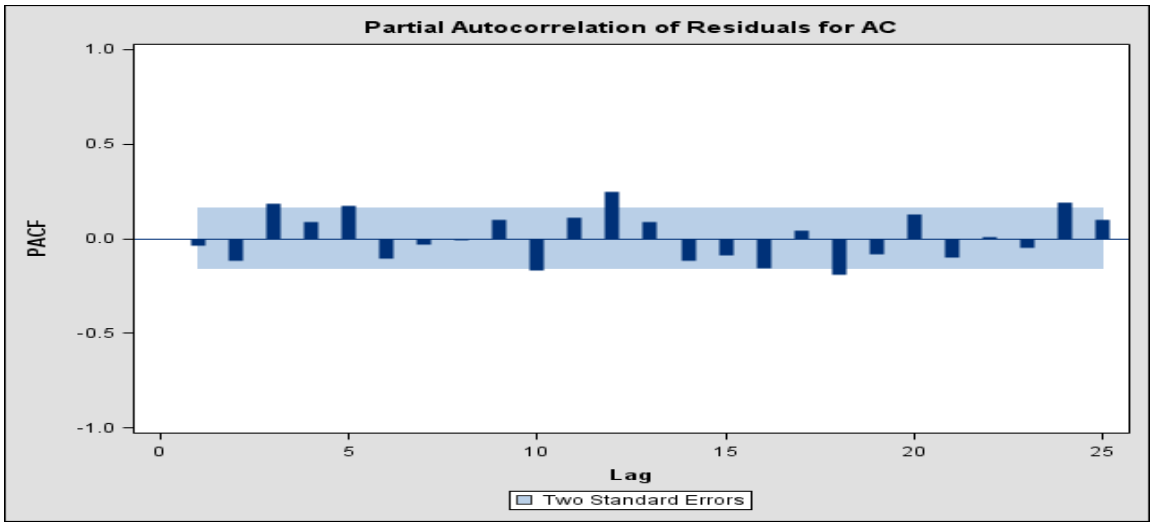


Figure 4.41 Partial autocorrelation of residuals plot for accumulators

4.2 Exponential smoothing

Exponential smoothing (ES) is a particular type of moving average technique applied to time series data. It is used to produce smoothed data for presentation, or to make forecasts. It describes a class of forecasting methods. ES assigns exponentially decreasing weights as the observations get older, i.e. new observations are given relatively more weights in forecasting than the older observations.

4.2.1 Types of Exponential smoothing

We describe three types of ES.

1. Single Exponential smoothing (SES)

SES, also known as simple exponential smoothing, is used for short-range forecasting, usually just one month into the future. This model assumes that the data fluctuates around a reasonably stable mean (no trend or consistent pattern of growth).

2. Double Exponential smoothing (DES)

DES method is used when the data shows a trend. Exponential smoothing with a trend works much like simple exponential smoothing except that two components must be updated each period level and trend. The level is the smoothed estimate of the value of the data at the end of each period and the trend is the smoothed estimate of the average growth at the end of each period.

3. Triple Exponential smoothing (TES)

TES method is used when the data shows trend and seasonality. To handle seasonality, a third parameter should be added and the resulting set of equations will be called the Holt-Winters. There are two main Holt-Winters models, depending on the type of seasonality. The methods are multiplicative seasonal model and additive seasonal model.

4.2.1.1 SES results: Accumulators

Statistical package “Zaitun Time Series” was used to select parameter values. Table 4.37 shows a summary of the results from the SES model. The only two accuracy measures of interest are MAE and MAPE. The smaller the accuracy measure the better the model. SES has the accuracy measures of MAE=11.84 and MAPE=11.96 values with $\alpha = 0.3$. Figure 4.42 shows the actual and predicted values of monthly data for the accumulators.

Table 4.37 SES model summary for the Accumulators

Alpha (for data)	0.300
Accuracy Measures	
Mean Absolute Error (MAE)	11.814404
Sum Square Error (SSE)	34598.201377
Mean Squared Error (MSE)	221.783342
Mean Percentage Error (MPE)	-1.113259
Mean Absolute Percentage Error (MAPE)	11.955545

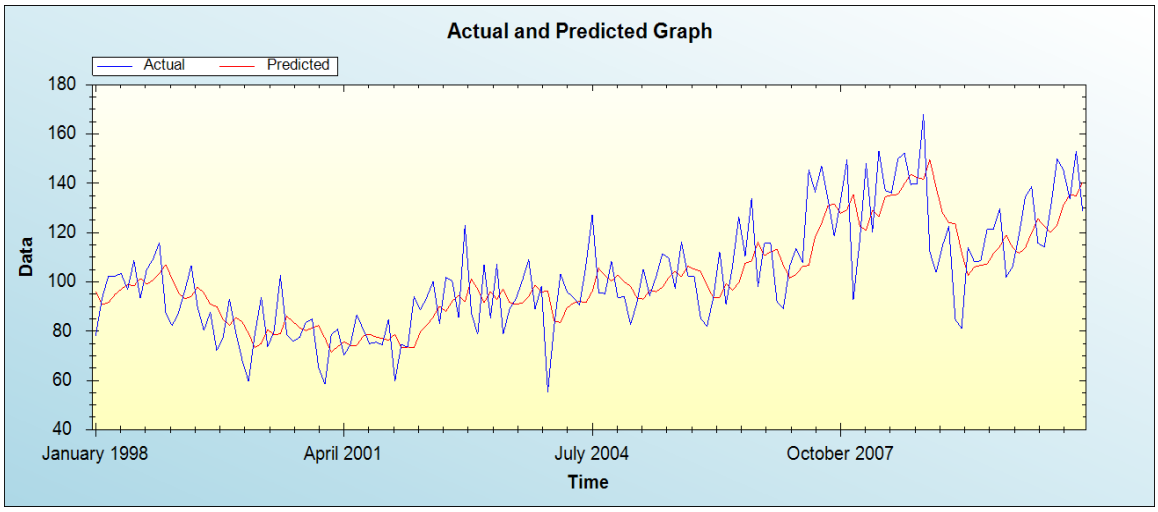


Figure 4.42 Actual and predicted accumulators graph using SES (Holt)

The plot of the residuals depicts a pattern that appears from the graph (figure 4.43). The next step is to check the DES and TES and determine which one has the best accuracy measures. This is done by selecting the one with the smallest accuracy measures from the three exponential smoothing models.

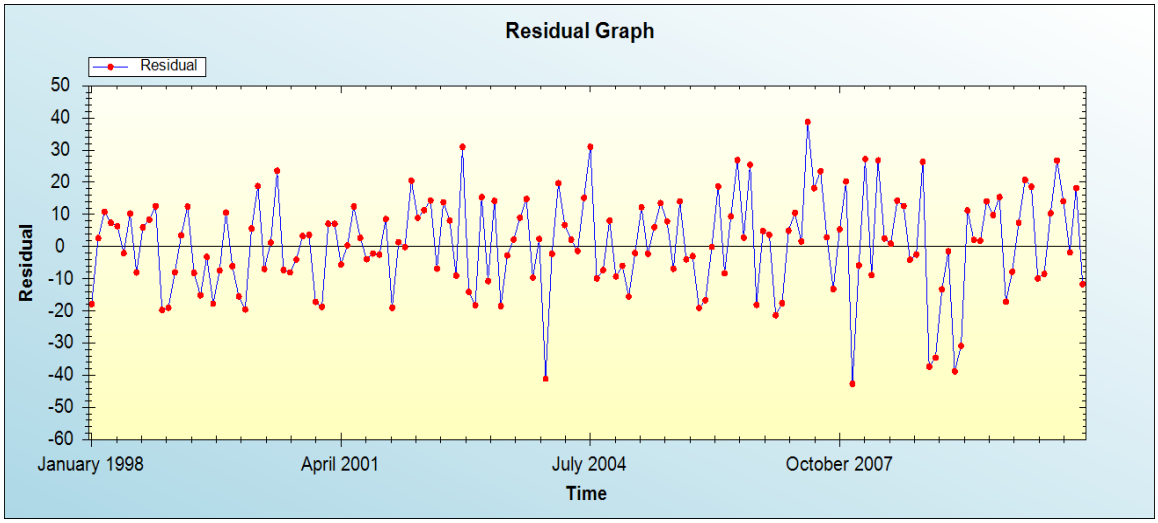


Figure 4.43 Residual graph for accumulators

4.2.1.2 DES results: Accumulators

Table 4.38 shows the results found from DES with α=0.4 and γ=0.1. SES does not appear to be the best model since the accuracy measures have higher values of MAE=12.45 and MAPE=12.65.

Table 4.38 DES model summary for accumulators

Alpha (for data)	0.400
Gamma (for trend)	0.100
Accuracy Measures	
Mean Absolute Error (MAE)	12.454235
Sum Square Error (SSE)	37594.129794
Mean Squared Error (MSE)	240.988011
Mean Percentage Error (MPE)	-1.514052
Mean Absolute Percentage Error (MAPE)	12.654135

Figure 4.44 shows the actual and predicted values of monthly data for the accumulators using DES method. The predicted values seem to follow the pattern formed by actual values of accumulators.

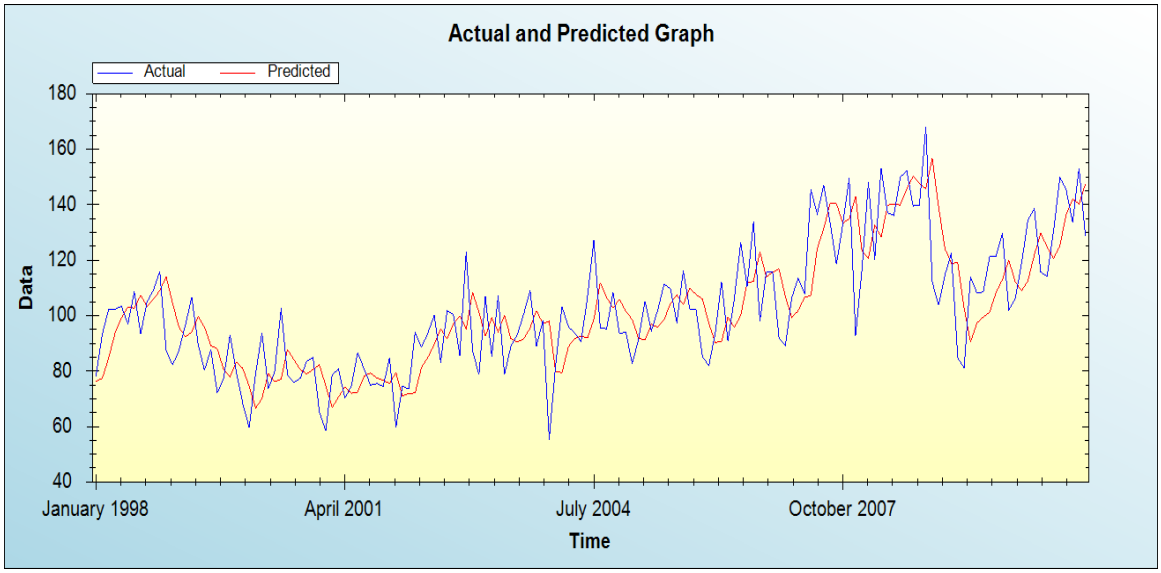


Figure 4.44 Actual and predicted graph for accumulators using DES (Holt)

Figure 4.45 presents the residual plots for the variable “accumulators”. The plot indicates that there is a pattern that appears from the graph. It is then required to assess the TES results and compare them with those from SES and DES.

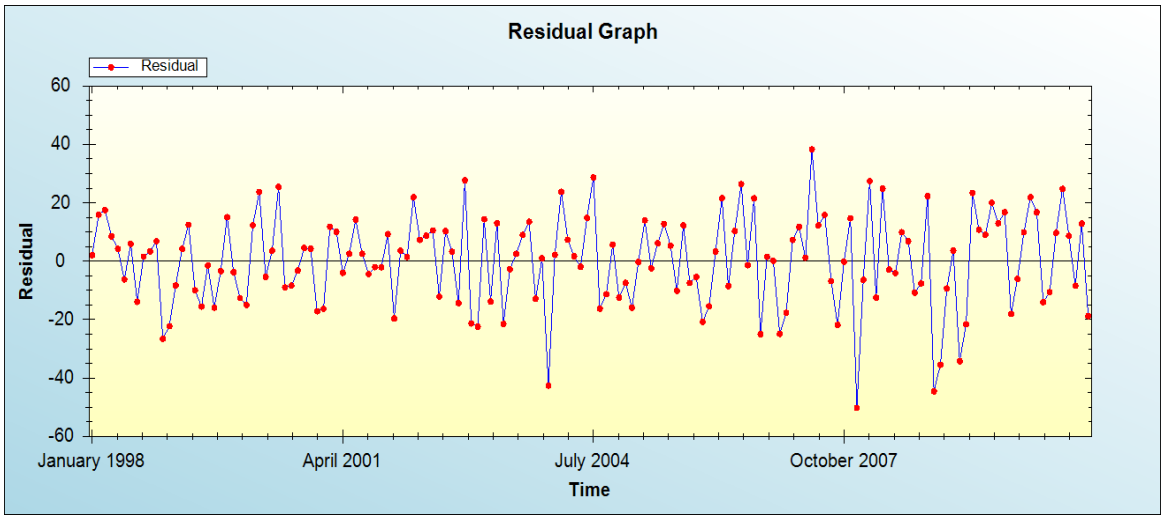


Figure 4.45 Residual graph for accumulators

4.2.1.3 TES results: Accumulators

Table 4.39 presents the results from TES with α=0.4, $\gamma = 0.1$ and β=0.1. SES and DES do not appear to be providing the best models because their values of accuracy measures are higher than those of the TES model, hence the TES model is the best model for this data generating smaller values of accuracy measures of MAE=9.80 and MAPE=9.72.

Table 4.39 TES model summary for accumulators

Alpha (for data)	0.4
Gamma (for trend)	0.1
Beta (for seasonal)	0.1
Accuracy Measures	
Mean Absolute Error (MAE)	9.801742
Sum Square Error (SSE)	24088.27023
Mean Squared Error (MSE)	154.411989
Mean Percentage Error (MPE)	-0.910708
Mean Absolute Percentage Error (MAPE)	9.716937

Figure 4.46 shows the actual and predicted values of monthly data for the accumulators. TES predicted the best results compared to SES and DES.

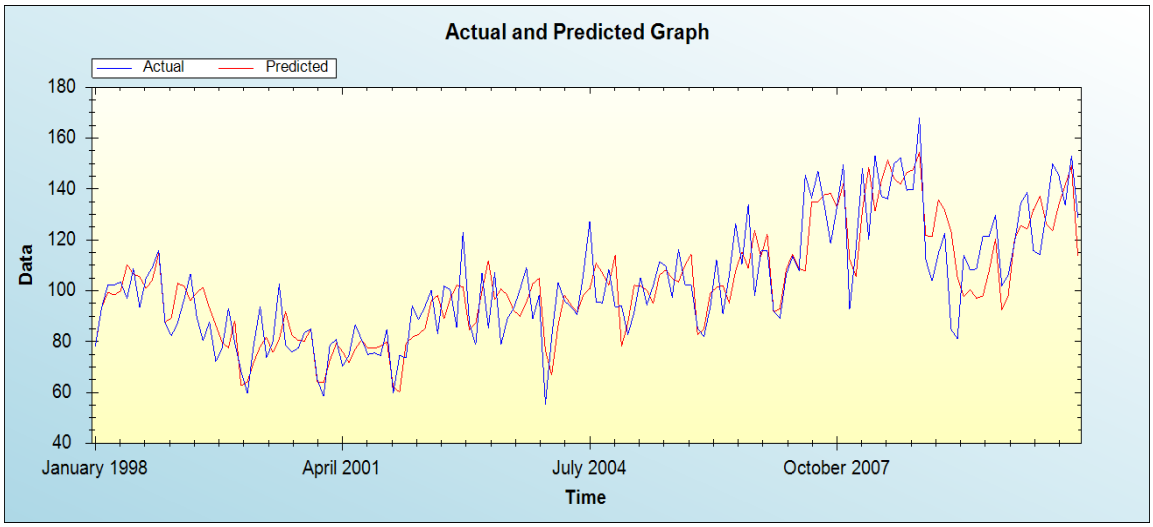


Figure 4.46 Actual and predicted graph for accumulators using TES (Holt)

The plot of the residuals (figure 4.47) indicates that there is no trend and the residuals are randomly fluctuating around zero.

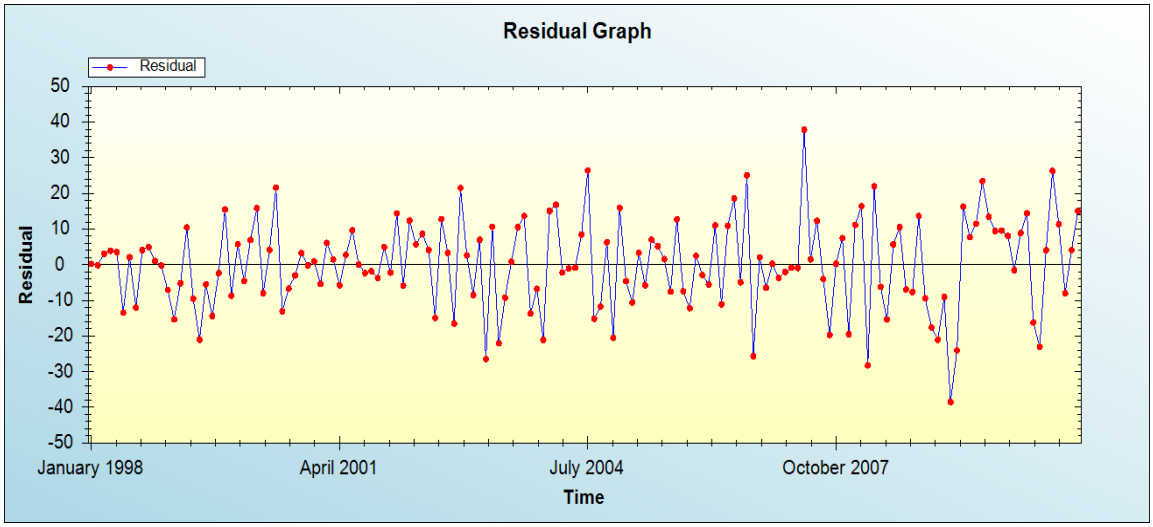


Figure 4.47 Residual graph for accumulators

4.2.1.4 Comparative model analysis: Accumulators

In this section, comparative analysis of the TES and neural networks model results is performed. The mean absolute error (MAE) and root mean square error (RMSE) are used for comparing these models. Table 4.40 presents the TES and neural networks with RMSE of 12.43 and 0.48, and MAE of 9.8 and 3.46, respectively. Since neural networks have the least values of MAE and RMSE, it is considered to be the best model to be fitted.

Table 4.40 TES and neural networks results: Accumulators

Performance criteria	Model	
	TES	Neural Networks
MAE	9.80	3.46
RMSE	12.43	0.48

Table 4.41 gives the short-term forecasted values using TES and neural networks for accumulators. Using the comparative analysis, neural networks outperformed the TES since it predicted the best model compared to SES and DES.

Table 4.41 One year forecast for accumulators

Period (1 Year)	Forecasting		
	TES	Neural Networks	Actual values
Jan-11	123.4120	148.7311	114.7
Feb-11	147.4224	150.1124	137.7
Mar-11	156.9043	130.8205	143.2
Apr-11	152.2763	154.6733	111.8
May-11	152.8114	104.6015	115.7
Jun-11	167.8487	128.3510	136.2
Jul-11	170.2942	103.7154	119.2
Aug-11	168.2151	128.2186	145.6
Sep-11	167.6442	95.1595	134.4
Oct-11	170.2876	52.7081	117.4
Nov-11	184.9422	57.5935	128.2
Dec-11	140.6878	78.2054	93.9

Figure 4.48 shows the graphical plot of actual and the predicted monthly values of the accumulators. The pattern of the actual values is observed, which has an indication that it can produce the best model compared to TES plot.

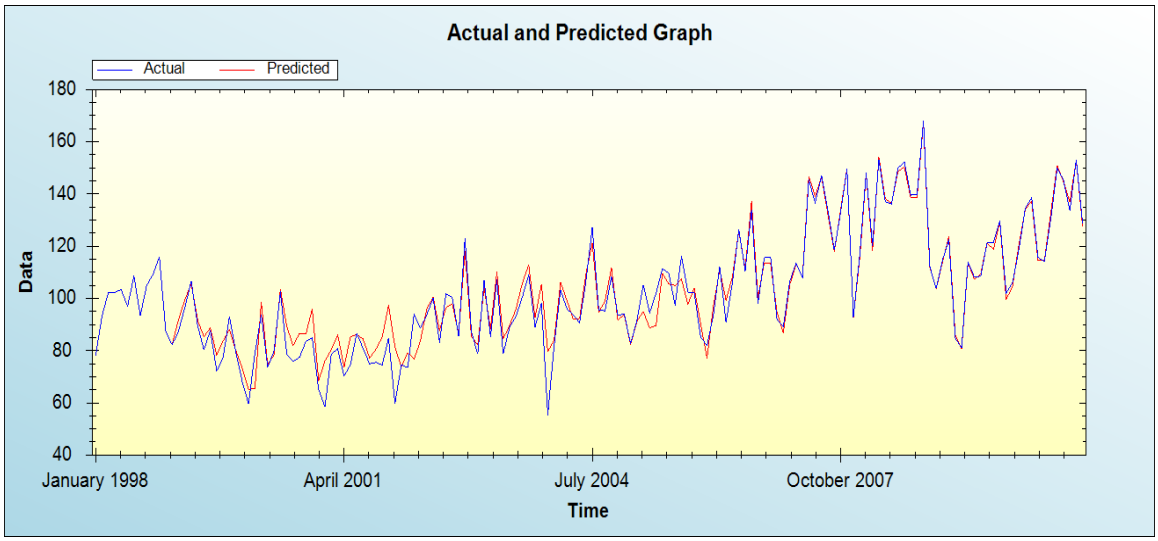


Figure 4.48 Actual and predicted graph for accumulators using neural networks

4.2.2.1 TES results: Lighting equipment

Table 4.42 indicates that the results from TES with α = 0.4, $\gamma = 0.1$ and β = 0.1 provide the best model compared to SES and DES (Appendix A2.2) since their values of accuracy measures were higher compared to the TES model. Thus TES model is the best model for this data with smaller accuracy measures of MAE and MAPE (8.53 and 8.36 respectively).

Table 4.42 TES model summary for lighting equipment

Alpha (for data)	0.4
Gamma (for trend)	0.1
Beta (for seasonal)	0.1
Accuracy Measures	
Mean Absolute Error (MAE)	8.53
Sum Square Error (SSE)	17033.84
Mean Squared Error (MSE)	109.19
Mean Percentage Error (MPE)	-1.03
Mean Absolute Percentage Error (MAPE)	8.36

Figure 4.49 shows the actual and predicted values of lighting equipment using TES. The plot shows that the predicted values seem to follow the seasonal pattern that is formed by the original values (actual values of lighting equipment). The results are then compared with the results generated by neural networks in table 4.43 and figure 4.44.

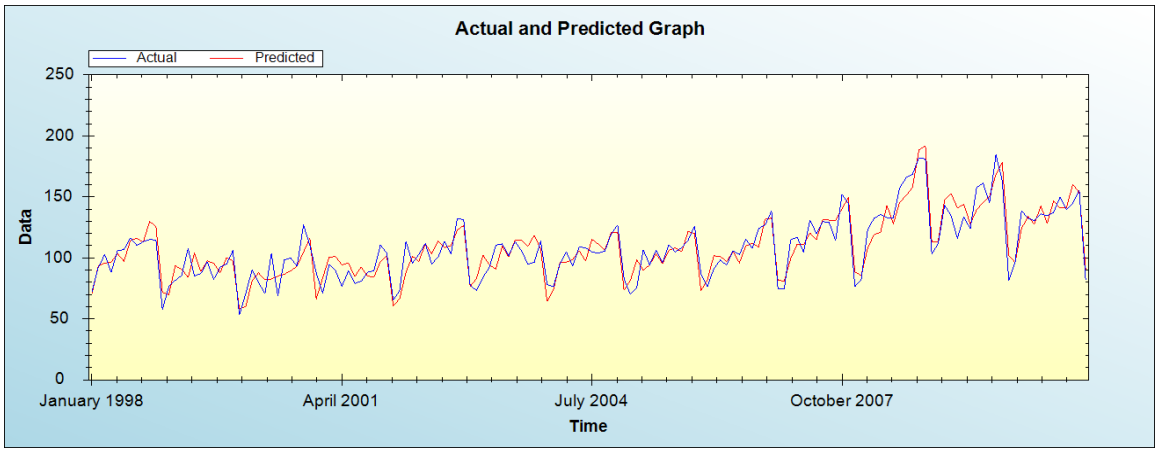


Figure 4.49 Actual and predicted graph for lighting equipment using TES (Holt)

Figure 4.50 shows the residual plot of lighting equipment, which indicates that there is no trend because the residuals are constant around mean zero.

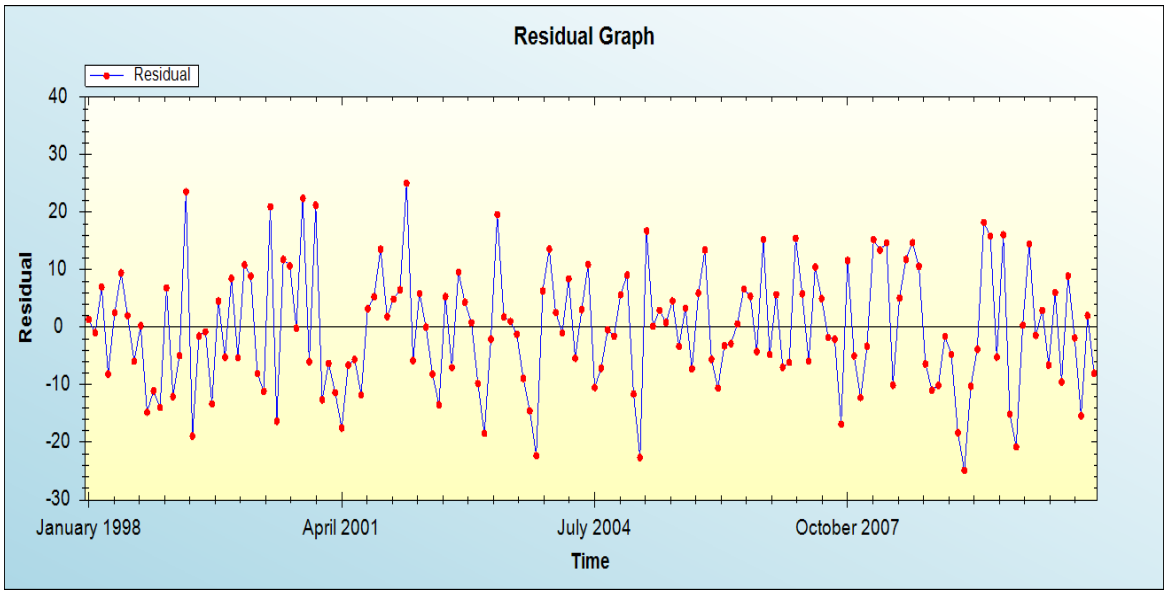


Figure 4.50 Residual graph for lighting equipment

4.2.2.2 Comparative model analysis: Lighting equipment

Comparative analysis of the ARIMA, TES and neural networks model results is performed. The RMSE and MAE of the three techniques is performed and neural networks was found to be the best predictor with smaller values of the errors, i.e. RMSE = 0.40 and MAE = 4.38.

Table 4.43 TES and neural networks results: Lighting equipment

Performance criteria	Model		
	TES	ARIMA	Neural Networks
MAE	8.530331	8.18	4.37663
RMSE	10.449460	10.52	0.40362

Table 4.44 shows both the forecasts from exponential smoothing, ARIMA and neural networks. The forecasted values to be considered are the ones which were created by neural networks since it generated minimum errors compared to TES.

Table 4.44 One year forecast for lighting equipment

Period (1 Year)	TES	ARIMA	Neural Networks	Actual values
Jan-11	90.1389	92.7884	69.5737	79.9
Feb-11	116.5828	122.1202	152.2121	133
Mar-11	118.2939	123.7899	163.1418	145.9
Apr-11	112.5606	118.7743	119.3162	126.3
May-11	123.1749	129.5221	102.3312	127.5
Jun-11	112.8458	122.8690	71.5500	147.8
Jul-11	124.5759	136.5953	64.6909	120.6
Aug-11	123.0088	139.3046	75.8854	157.1
Sep-11	118.7897	134.7883	94.3615	148.6
Oct-11	133.6877	151.9751	129.5268	168
Nov-11	133.5377	152.7876	169.2942	177.3
Dec-11	77.2135	89.8782	53.3467	81.4

Figure 4.51 shows the graphical plot of actual and the predicted monthly values of the lighting equipment. The predicted values follow the pattern of the actual values as an indication that it can produce the best model compared to TES plot.

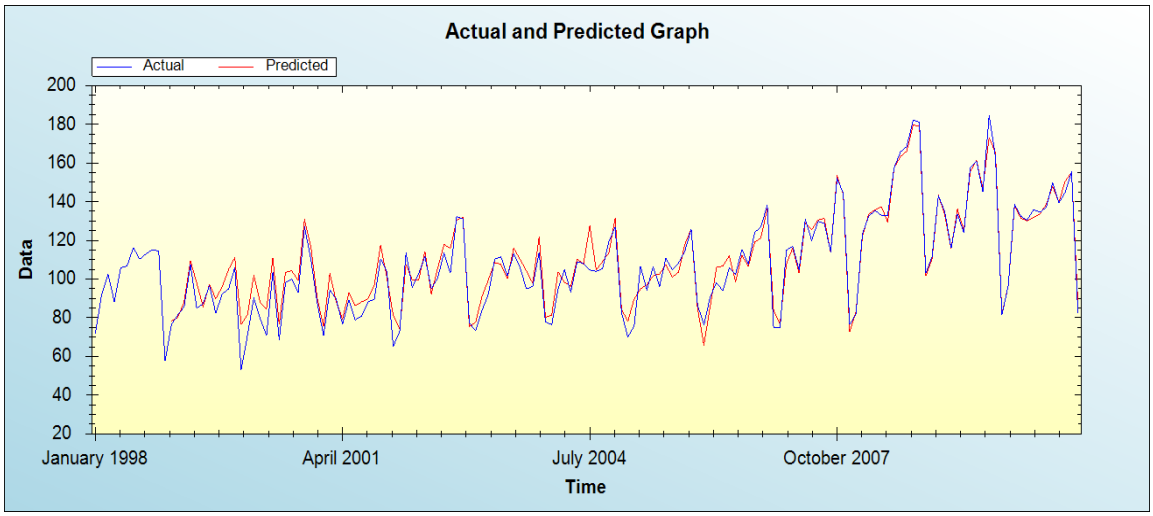


Figure 4.51 Actual and predicted graph for lighting equipment using neural networks

Figure 4.52 shows the residual plot of lighting equipment and indicates that there is no trend because the residuals are constant around mean zero.

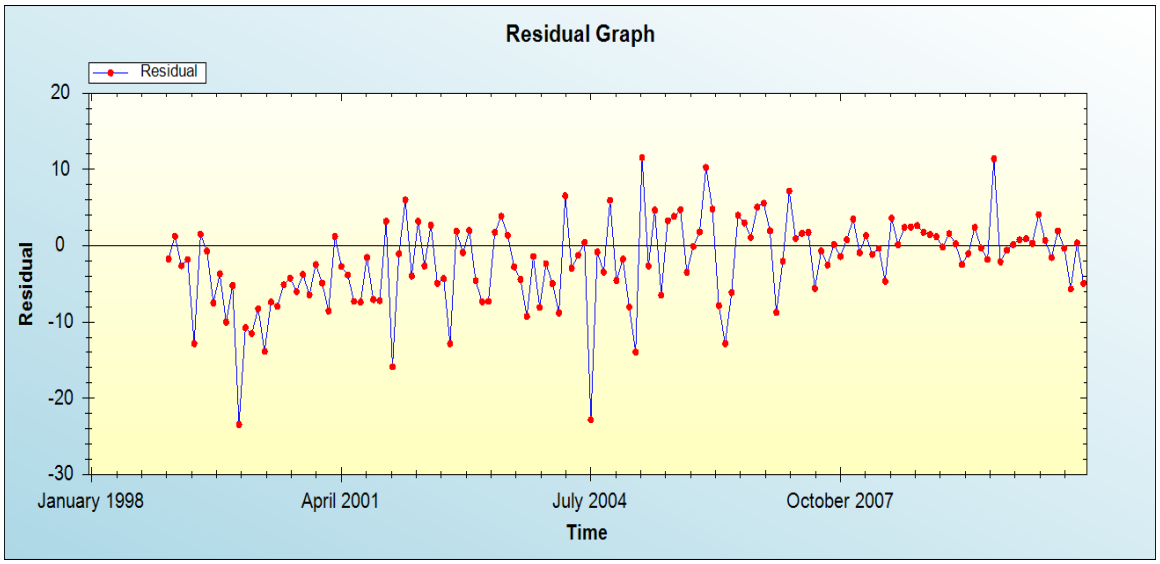


Figure 4.52 Actual and predicted graph for lighting equipment using neural networks

4.2.3.1 TES results: Electric machines

Table 4.45 shows the results found from TES with α=0.3, $\gamma = 0.1$ and β=0.1. TES provides the best model compared to SES and DES (Appendix A2.3) since the values of accuracy measures for SES and DES were higher compared to the TES model. TES model is the best model for electric machines data since it has the smaller accuracy measures of MAE and MAPE which are 5.01 and 4.57 respectively.

Table 4.45 TES model summary for lighting equipment

Alpha (for data)	0.3
Gamma (for trend)	0.1
Beta (for seasonal)	0.1
Accuracy Measures	
Mean Absolute Error (MAE)	5.01205
Sum Square Error (SSE)	6315.581
Mean Squared Error (MSE)	40.48449
Mean Percentage Error (MPE)	-0.23336
Mean Absolute Percentage Error (MAPE)	4.56679

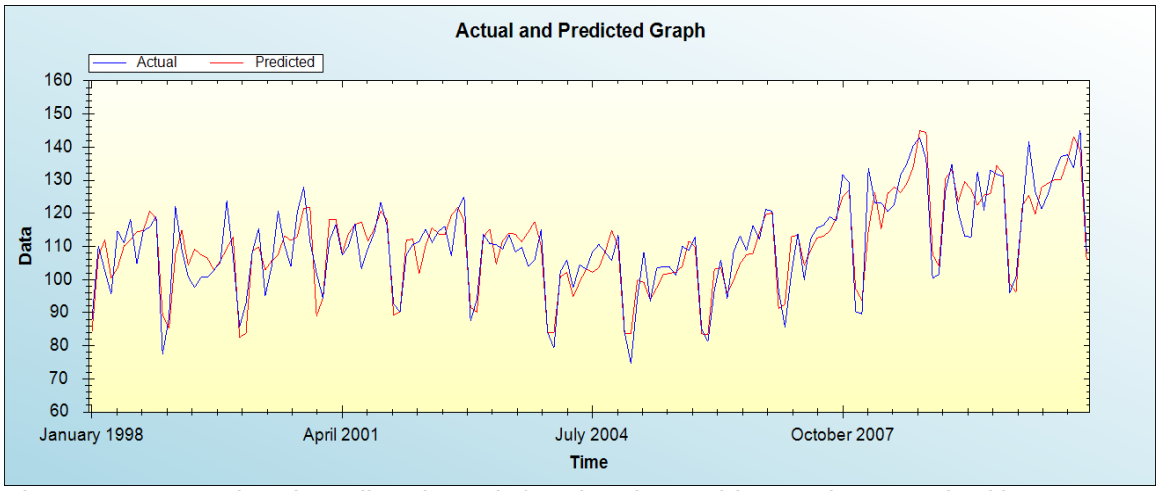


Figure 4.53 Actual and predicted graph for electric machines using TES (Holt)

Figure 4.54 shows the residual plot of electric machines and indicates that there is no trend because the residuals are constant around mean zero.

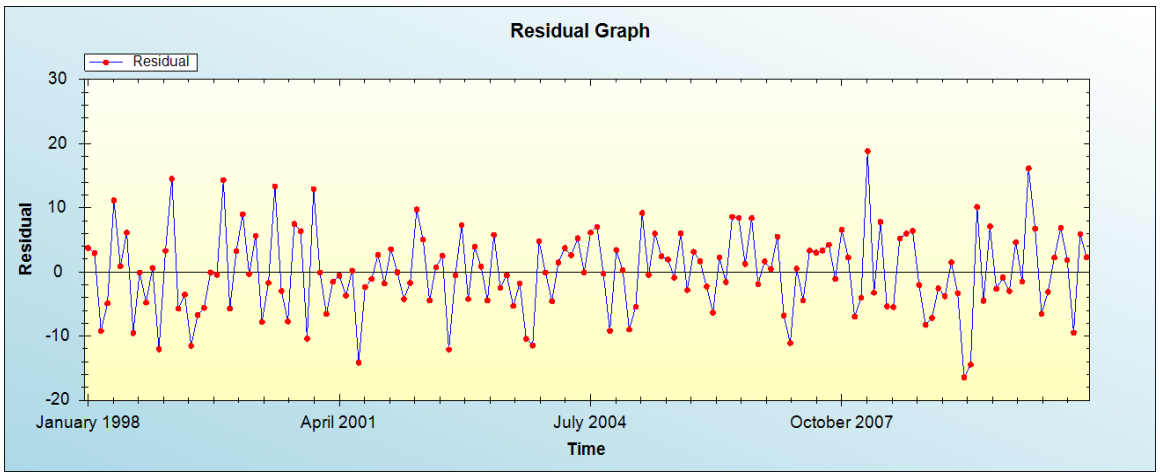


Figure 4.54 Residual graph for electric machines using TES

4.2.3.2 Comparative model analysis: Electric machines

Comparative analysis of the TES, ARIMA and neural networks model results is performed in Table 4.46 by comparing the RMSE and MAE for the three techniques. Neural Networks was found to be the best predictor of the model by having smaller values of the errors i.e. 0.15 and 1.31.

Table 4.46 TES and neural networks results: Electric machines

Performance criteria	Model		
	TES	ARIMA	Neural Networks
MAE	5.012058	4.48266	1.312039
RMSE	6.362742	5.85547	0.147486

Table 4.47 shows both the forecasts from ARIMA, TES and neural networks. The forecasted values to be considered are the ones which were created by neural networks since it is the one with minimum errors compared to TES.

Table 4.47 One year forecast for electric machines

Period (1 Year)	TES	ARIMA	Neural networks	Actual values
Jan-11	106.1498	108.1551	97.9053	106.9
Feb-11	133.2847	123.0454	123.7332	132.0
Mar-11	139.1603	134.0101	136.1493	144.7
Apr-11	127.4952	121.6860	116.0232	118.5
May-11	132.8614	124.8518	126.4274	126.0
Jun-11	136.6502	128.2026	109.3370	132.9
Jul-11	139.4348	125.1576	134.8024	122.0
Aug-11	139.4816	129.9283	127.4171	133.3
Sep-11	142.8753	127.7204	132.2890	152.1
Oct-11	149.0648	129.7977	140.2567	139.6
Nov-11	149.2470	136.2427	143.6134	153.3
Dec-11	112.2193	105.3092	87.7596	98.6

Figure 4.55 indicates the graph of electic machines and the predicted plot using neural networks. Neural networks plot (predicted plot) shows that it is indeed the best since it is following the pattern of the actual values of electric machines.

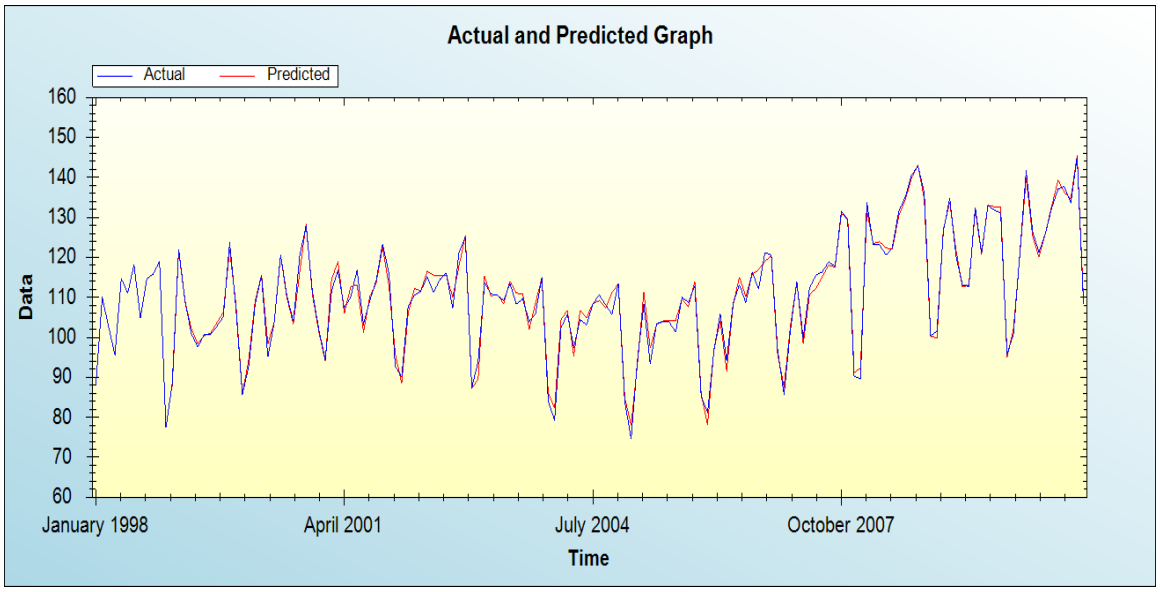


Figure 4.55 Actual and predicted graph for electric machines using neural networks

Figure 4.56 shows the residual plot of electric machines and the figure indicates that there is no pattern formed because the residuals are constant around mean zero.

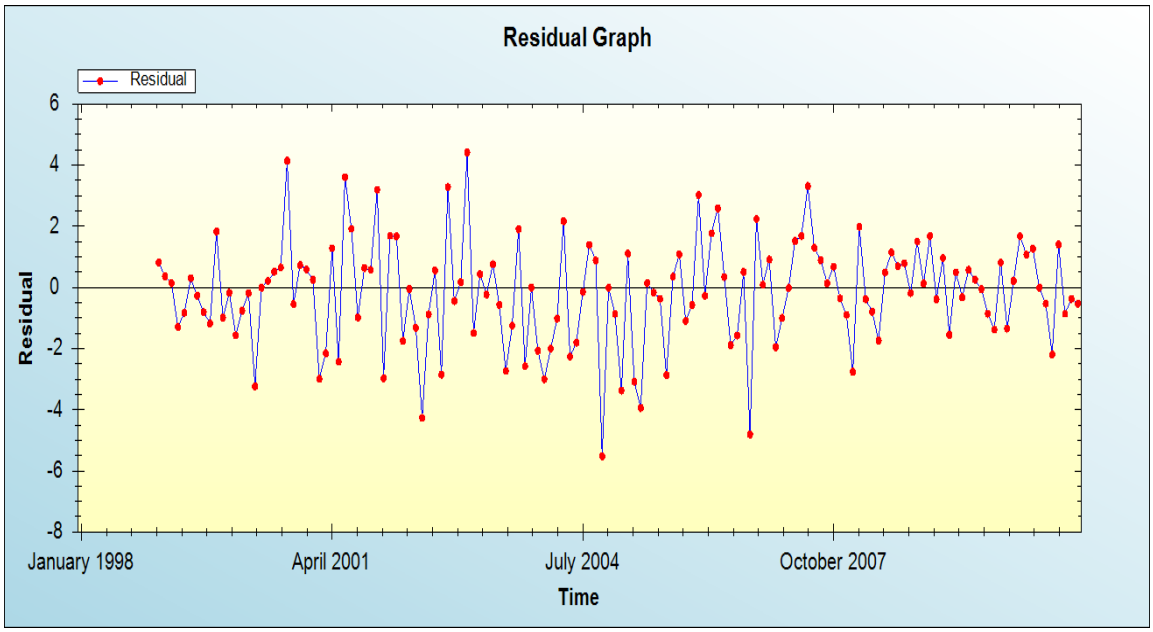


Figure 4.56 Residual graph for electric machines using neural networks

4.2.4.1 TES results: Communication apparatus

The results in Table 4.48 were generated from TES with α=0.5, $\gamma = 0.1$ and β=0.1. TES is the best model compared to SES and DES (Appendix A2.1) since their values of accuracy measures were higher compared to those of the TES model. TES model is the best model for

this data with smaller accuracy measures of MAE and MAPE, which are 9.06 and 9.67, respectively.

Table 4.48 TES model summary: Communication apparatus

Alpha (for data)	0.5
Gamma (for trend)	0.1
Beta (for seasonal)	0.1
Accuracy Measures	
Mean Absolute Error (MAE)	9.064287
Sum Square Error (SSE)	21226.39
Mean Squared Error (MSE)	136.06660
Mean Percentage Error (MPE)	-1.17068
Mean Absolute Percentage Error (MAPE)	9.67242

Figure 4.57 shows the actual values of communication apparatus from TES that fitted well compared to SES and DES.

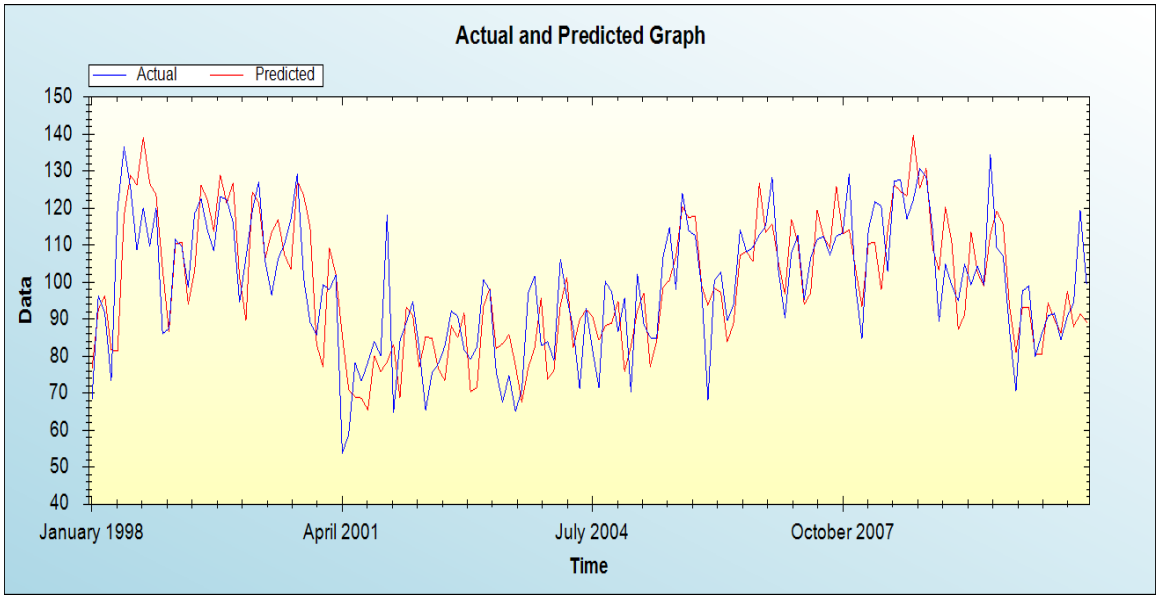


Figure 4.57 Actual and predicted graph for communication apparatus using TES

Figure 4.58 shows the residual plot of communication apparatus, and indicates that there is no trend because the residuals are constant around mean zero.

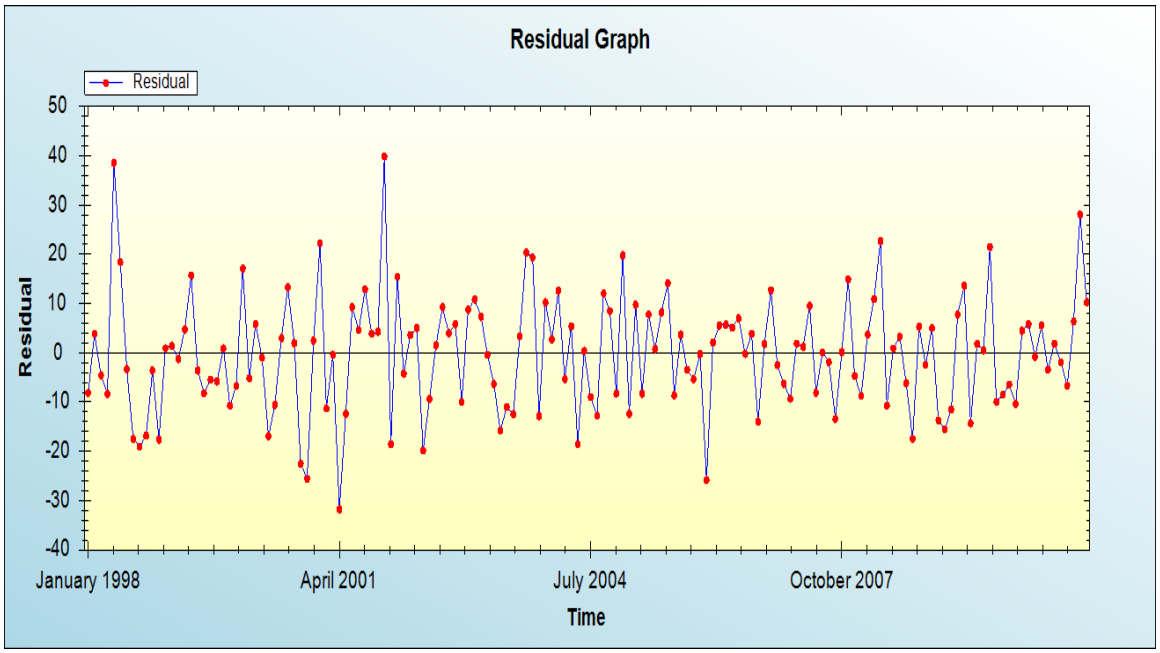


Figure 4.58 Residual graph for communication apparatus using TES

4.2.4.2 Comparative model analysis: Communication apparatus

Comparative analysis of the TES and neural networks model results is performed. Table 4.49 presents the TES and neural networks results. The TES model has MAE and RMSE of 9.06 and 11.66, respectively; and neural networks model has MAE and RMSE of 1.53 and 0.18, respectively. Since neural networks model has the small values of MAE and RMSE it is considered to be the best model to be fitted.

Table 4.49 TES and neural networks results: Communication apparatus

	Model	
Performance criteria	TES	Neural Networks
MAE	9.0642	1.532061
RMSE	11.6600	0.184373

Table 4.50 indicates the short-term forecasted values using TES and neural networks for communication apparatus. Using the comparative analysis, neural networks outperformed the TES.

Table 4.50 One year forecast for communication apparatus

Period (1 Year)	Forecasting		
	TES	Neural Networks	Actual values
Jan-11	86.0517	91.0477	61.8
Feb-11	109.5270	114.0435	119.9
Mar-11	109.5417	114.5475	93.8
Apr-11	93.6122	105.4487	77.3
May-11	96.0300	123.2062	92
Jun-11	110.2820	134.0027	97.4
Jul-11	109.1477	125.2421	114
Aug-11	105.6018	76.9848	121.1
Sep-11	122.8064	105.1858	145.3
Oct-11	118.4200	118.7969	117.2
Nov-11	122.2045	129.8764	123.9
Dec-11	102.9943	55.2551	86.1

Figure 4.59 shows the graphical plot of actual values of the communication apparatus and the predicted monthly values of communication apparatus using neural networks. The predicted graph shows a pattern of the actual values which is a reflection that neural networks produced the best model compared to TES plot.

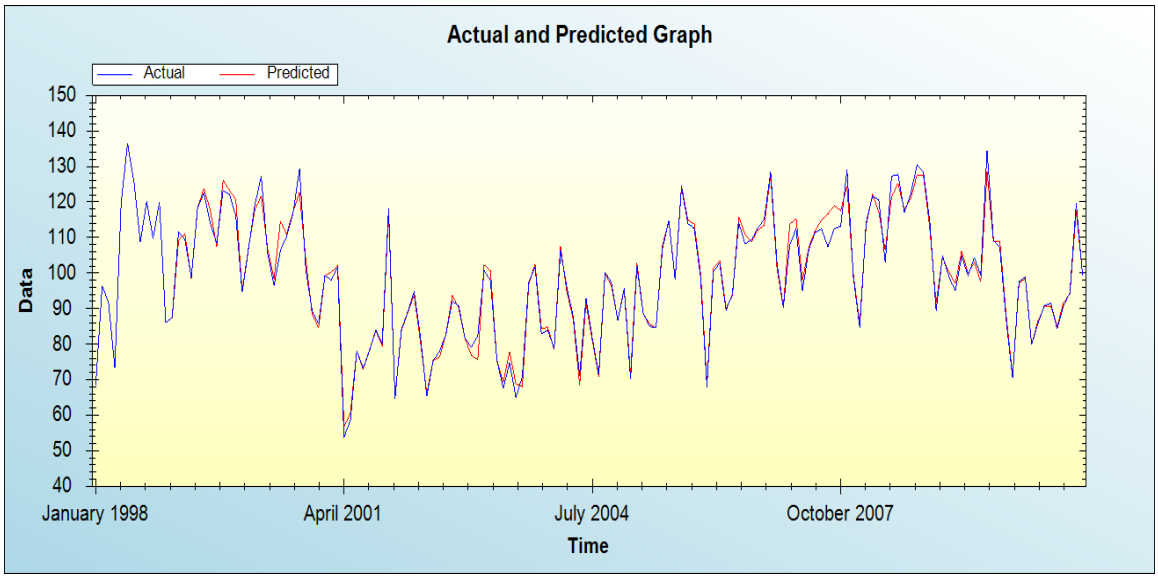


Figure 4.59 Actual and predicted graph for communication apparatus using neural networks

Figure 4.60 shows the residual plot of communication apparatus which indicates that there is no trend because the residuals are constant around mean zero.

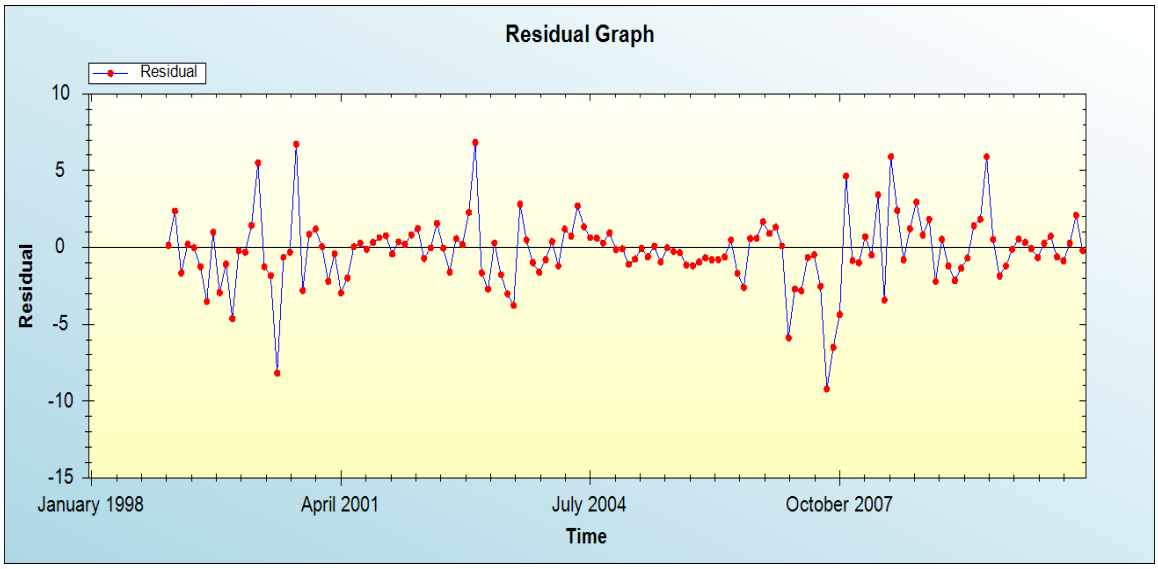


Figure 4.60 Residual graph for communication apparatus using neural networks

4.2.5.1 TES results: Other electrical equipment

Table 4.51 shows the results generated from TES with α=0.2, $\gamma = 0.1$ and β=0.1. TES provides the best model compared to SES and DES (Appendix A2.4) since the values of accuracy measures for SES and DES were higher compared to the TES model. TES model is the best model for this data with smaller accuracy measures of MAE and MAPE which are 8.33 and 10.44 respectively.

Table 4.51 TES model summary for other electrical equipment

Alpha (for data)	0.2
Gamma (for trend)	0.1
Beta (for seasonal)	0.1
Accuracy Measures	
Mean Absolute Error (MAE)	10.43655
Sum Square Error (SSE)	28873.47193
Mean Squared Error (MSE)	185.08636
Mean Percentage Error (MPE)	-0.41479
Mean Absolute Percentage Error (MAPE)	8.33371

Figure 4.61 shows the actual and predicted values of communication apparatus from TES. TES fitted well by following the pattern of the actual values of other electrical equipment.

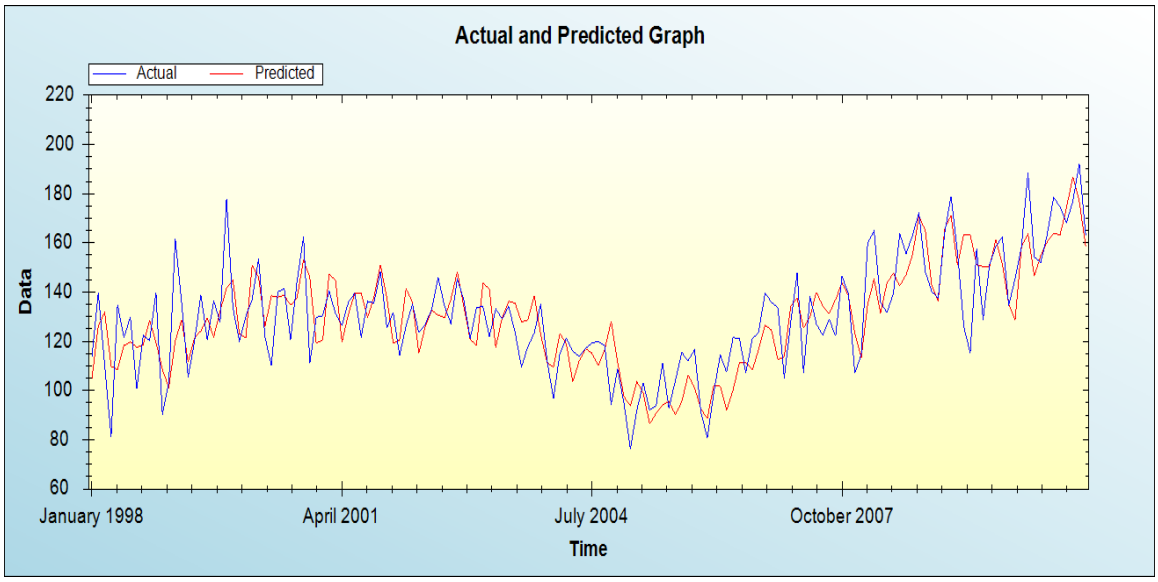


Figure 4.61 Actual and predicted graph for other electrical equipment using TES

Figure 4.62 presents the residual plot of other electrical equipment which reveals that there is no trend because the residuals are constant around mean zero.

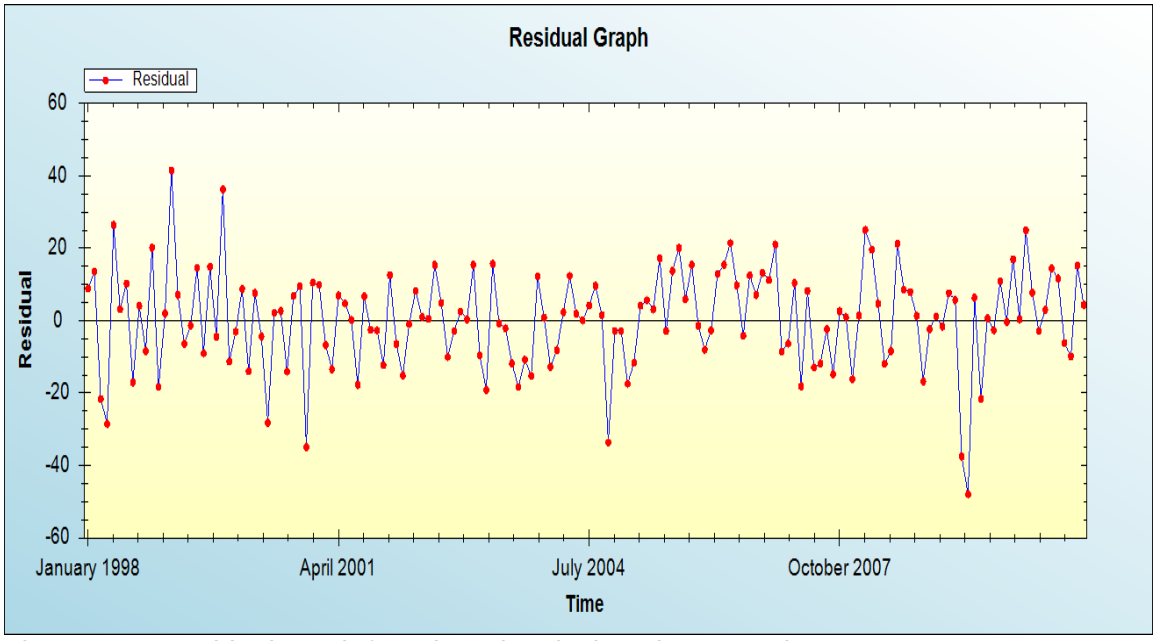


Figure 4.62 Residual graph for other electrical equipment using TES

4.2.5.2 Comparative model analysis: Other electrical equipment

Comparative analysis of the TES, ARIMA and neural networks model results is performed. Table 4.52 shows the comparative analysis of TES, ARIMA and the results from neural networks. TES and neural networks models have RMSE of 13.60, 13.24 and 0.32 respectively and MAE of 10.44, 10.05 and 3.03 respectively. Since neural networks has the small values of MAE and RMSE, it is considered to be the best model for forecasting prices of other electric equipment.

Table 4.52 TES and neural networks results: Other electrical equipment

	Model		
Performance criteria	TES	ARIMA	Neural Networks
MAE	10.44	10.05	3.03
RMSE	13.60	13.24	0.32

Table 4.53 presents the short-term forecasted monthly values for 2011 using TES, ARIMA and neural networks for other electrical equipment. Using the comparative analysis, neural networks outperformed the TES and ARIMA as the best predictive model. Forecasted values from neural networks will be considered since they have the smaller values of RMSE and MAE.

Table 4.53 One year forecast for other electrical equipment

	Forecasting			
Period (1 Year)	TES	ARIMA	Neural Networks	Actual values
Jan-11	155.0360	162.9260900	104.4897	151.7
Feb-11	185.5668	184.3282224	89.6304	181.2
Mar-11	195.1242	196.0198406	194.0351	183.7
Apr-11	168.8430	173.1315525	134.8331	143.6
May-11	176.1188	177.1312691	194.1389	158.5
Jun-11	184.1556	184.0103967	104.0365	155.9
Jul-11	189.0652	190.9913399	195.2032	149.3
Aug-11	184.7671	187.7747437	187.6238	140.5
Sep-11	192.7898	190.4944466	195.4617	163.2
Oct-11	207.9910	200.8999792	156.9773	156
Nov-11	201.5963	199.3895641	197.7333	188.1
Dec-11	176.7968	179.234547	187.0586	118.5

Figure 4.63 shows the actual and predicted values of other electrical equipment using neural networks, and reveals that the actual and predicted values seem to follow the same pattern.

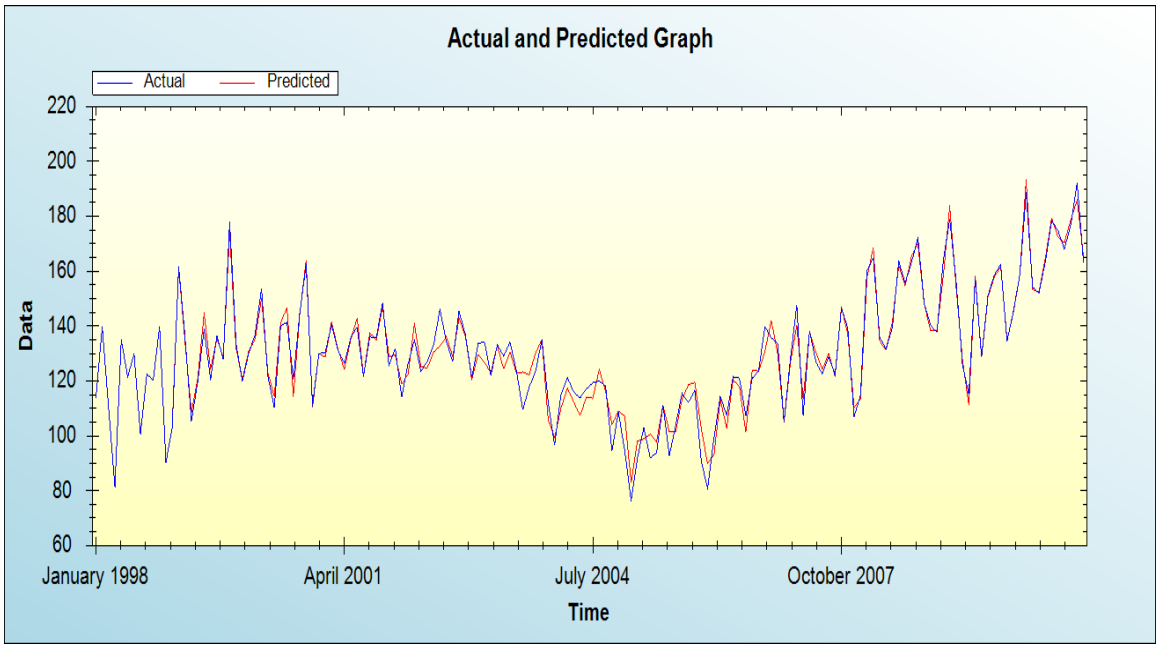


Figure 4.63 Actual and predicted graph for other electrical equipment using neural networks

Figure 4.64 shows the residual plot of other electrical equipment, and reveals that there is no trend because the residuals are constant around mean zero.

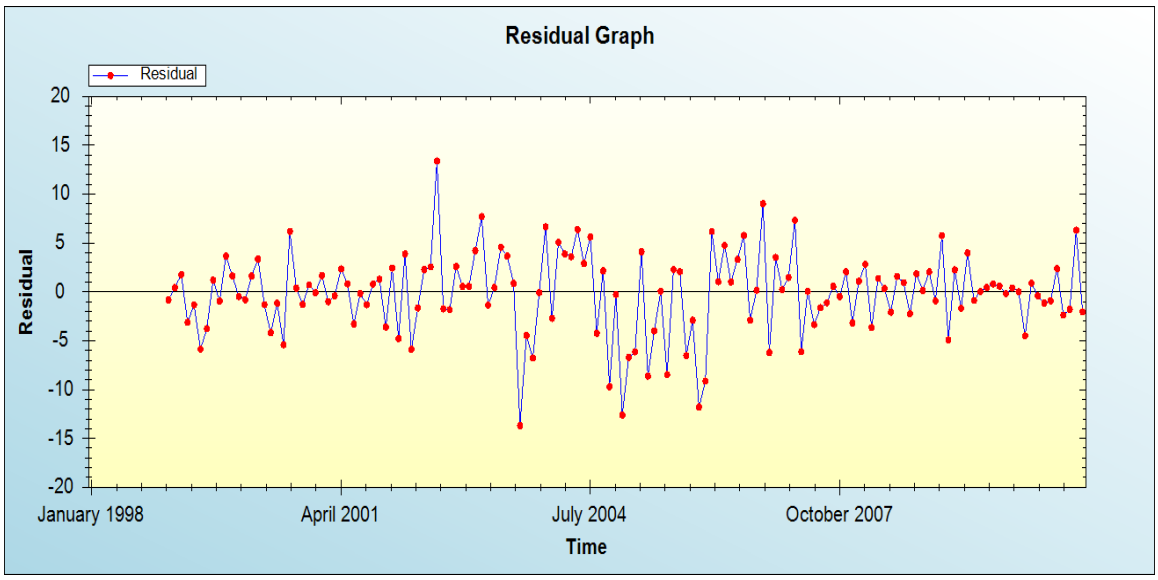


Figure 4.64 Residual graph for other electrical equipment using neural networks

4.3 Regression analysis

Regression analysis is a statistical technique investigating relationships between variables. Usually the researcher seeks to ascertain the impact of a number of (independent) variables on a single (dependent) variable. The general formulae for multiple linear regression model can be written as the response of y (dependent variable) to several predictors (independent variables), i.e.

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k + \varepsilon \quad \text{(3.109)}$$

where y is the dependent variable (response variable. The parameters $\beta_0, \beta_1, \beta_2, \ldots, \beta_k$ are regression coefficients and $x_1, x_2, x_3, \ldots, x_k$ are the predictors (independent variables).

In this study regression analysis is applied in assessing the effect of price indices of electrical appliances. In order to incorporate this effect in regression model, monthly dummy variables were created. The number of variables depend on the type of data, 11 (eleven) dummy variables are created on a 12-month basis. The rule in regard to dummy variables is that the number of dummy variables should be equal to the number of categories minus one (Levine *et al*., 2003). The regression model for this study is written as:

$$\text{LQ} = \sum_{i=1}^{12} d_i + t \quad \text{(3.110)}$$

where LQ (lighting equipment) is used as predicted variable, d_i represents the dummies for the month effect and t represents trend line or time.

The model was fitted using the transformed LQ (lighting equipment) data after removing one of the independent variables (*d2).* Since *d2* was found not to be significant we then reran the data and the equation developed without that variable.

Table 4.54 shows the model coefficients for the transformed LQ data. The significant column (sig.) indicates a significant relationship in linear trend (sig. = 0.000). It is also noticed that all the independent variables are significant predictors at the 5% level. The full regression equation using unstandardised coefficients (B) is given by:

LQ (lighting equipment)=*0.014-0.000027*trend+*0.001d1-0.002d3-0.002d4-0.003d5-0.002d6 -0.003d7-0.003d8-0.003d9-0.004d10-0.004d11*

where

d1=January

d2=February

d3=March

d4=April

d5=May

d6=June

d7=July

d8=August

d9=September

d10=October

d11=November

d12=December

t=trend

Table 4.54 Model coefficients for LQ

Coefficients[a]					
	Unstandardised Coefficients		Standardised Coefficients		
Model 1	B	Std. Error	Beta	t	Sig.
(Constant)	.014	.000		40.350	.000
Trend	-2.734E-05	.000	-.518	-10.785	.000
d1	.001	.000	.123	2.180	.031
d3	-.002	.000	-.241	-4.278	.000
d4	-.002	.000	-.191	-3.399	.001
d5	-.003	.000	-.311	-5.532	.000
d6	-.002	.000	-.208	-3.700	.000
d7	-.003	.000	-.330	-5.871	.000
d8	-.003	.000	-.320	-5.696	.000
d9	-.003	.000	-.304	-5.414	.000
d10	-.004	.000	-.425	-7.569	.000
d11	-.004	.000	-.441	-7.844	.000
a. Dependent Variable: LQ_transformed					

Table 4.55 presents the model summary. The R-square for the regression model indicates that 66.9% of the variation in the price of lighting equipment is accounted for by the regression model. The adjusted R-square corrects for the fact that in regression models the R-square increases with the number of independent variables or predictors (*d1, d3, … ,d11* and t). The adjusted R-square estimates the explained model variation more predictably at 64.4%, which seems reasonably high.

Table 4.55 Model summary for LQ transformed

Model Summary[b]					
Model1	R	R Square	Adjusted R Square	Std. Error of the Estimate	Durbin-Watson
	.818[a]	.669	.644	.0014232	1.432
a. Predictors: (Constant), d11, Trend, d6, d7, d5, d8, d4, d9, d3, d10, d1					
b. Dependent Variable: LQ_transformed					

The hypotheses tested for this model are:

H_O: There is no linear relationship between the dependent variable (LQ) and the predictors or independent variables.

H_1: There is a linear relationship between the dependent variable (LQ) and the predictors or independent variables.

Table 4.56 provides the ANOVA table results. A p-value of 0.000 in table 4.56 indicates that at least one of the independent variables or predictors is significant in predicting the price indices of LQ (lighting equipment) at 5% significant level.

Table 4.56 ANOVA table

ANOVA[a]					
Model 1	Sum of Squares	df	Mean Square	F	Sig.
Regression	.001	11	.000	26.476	.000[b]
Residual	.000	144	.000		
Total	.001	155			
a. Dependent Variable: LQ_transformed					
b. Predictors: (Constant), d11, Trend, d6, d7, d5, d8, d4, d9, d3, d10, d1					

Figure 4.65 fits regression model to the transformed LQ data. The interest is to find how well the model fits the data. The R-square indicates that the model fit is good (R-square=0.669), but this needs to be confirmed by plotting lighting equipment and unstandardised predicted variables. The fit looks fairly impressive, although the model sometimes tends to over predict and under predict at the peaks.

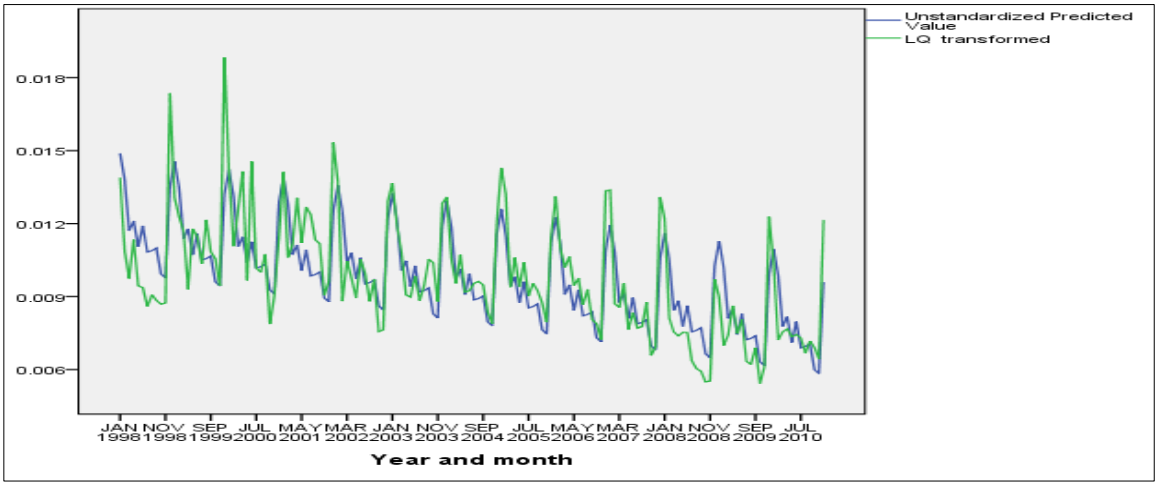


Figure 4.65 Fitting regression model to LQ data

Figure 4.66 is the sequence charts of errors of transformed LQ data. The figure intends to find out if the errors are random. There appears to be a roughly approximately equal number of positive and negative errors. The errors are not random because one positive error tends to be followed by another. This indicates that there might be autocorrelation left.

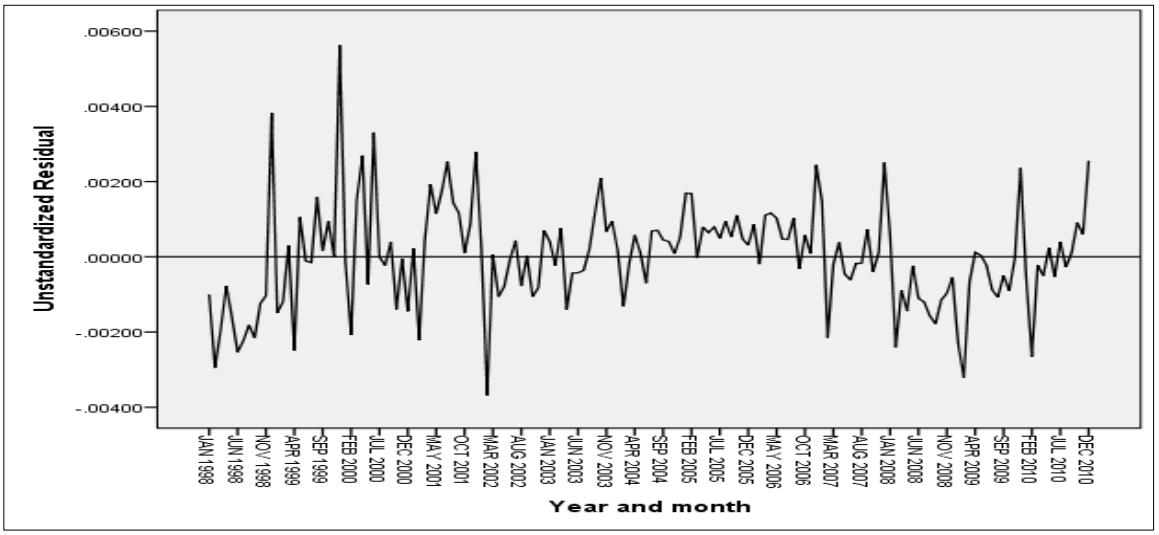


Figure 4.66 The sequence charts of errors

Figure 4.67 is the ACF plot for transformed LQ data. It can be deduced from ACF plot that there is evidence of autocorrelation. In fact, there is a tendency of positive errors but decaying slowly. Most of the lags attain significance. For example, for lag 1, lag 3 and lag 12 and only three lags are significant.

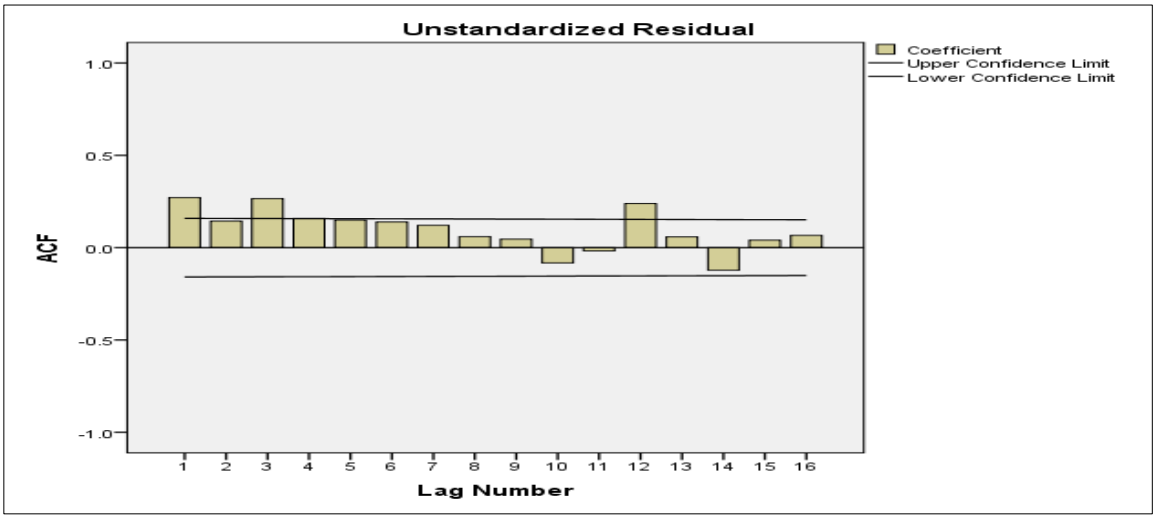


Figure 4.67 ACF plot of regression error

Figure 4.68 shows the unstandardised residual for the transformed LQ data. The PACF plot has basically the same pattern but for only 3 lags being significant (lag 1, lag 3 and lag 12).

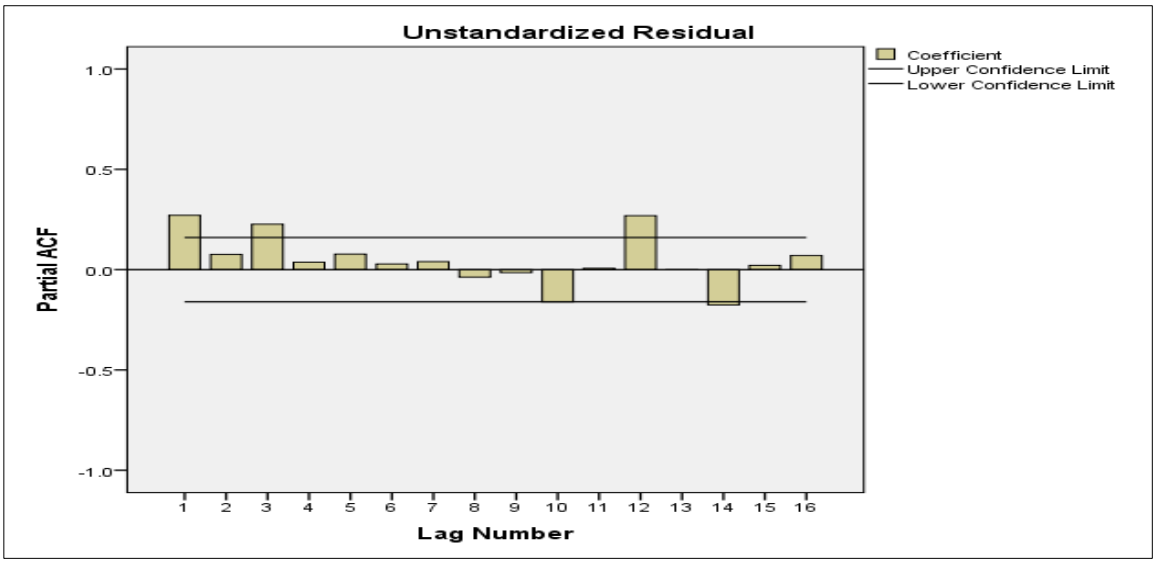


Figure 4.68 The PACF plot of regression error

4.4 Multivariate time series

4.4.1 Summary statistics

Table 4.57 represents the summary statistics for the five variables, namely: Lighting equipment, Communication apparatus, Electric machines, Accumulators and Other electrical equipment. The table also shows the minimum and the maximum values of each variable of the price indices of electrical appliances.

Table 4.57 Simple Summary Statistics

Simple Summary Statistics							
Variable	Type	N	Mean	Standard Deviation	Min	Max	Label
LQ	Dependent	156	107.85128	26.07808	53.10	184.50	LIGHTING EQUIPMENT
CA	Dependent	156	98.83910	17.57215	53.80	136.50	COMMUNICATION APPARATUS
EM	Dependent	156	110.61346	14.24103	74.70	145.10	ELECTRIC MACHINES
AC	Dependent	156	102.28333	22.89844	55.40	168.00	ACCUMULATORS
OEE	Dependent	156	130.15769	22.31377	76.30	192.20	OTHER ELECTRICAL EQUIPMENT

4.4.2 Unit Root Test

Dickey-Fuller unit root test is the most widely used test for non-stationarity that was developed by Dickey and Fuller (1979).

Table 4.58 presents the Dickey–Fuller Root Test and it is used to check whether or not the variables are stationary. Table 4.58 suggests that the variables attain stationarity except only AC with the single mean of 0.0113.

Table 4.58 Dickey-Fuller Unit Root Tests

Dickey-Fuller Unit Root Tests					
Variable	Type	Rho	Pr < Rho	Tau	Pr < Tau
LQ	Zero Mean	-2.19	0.3083	-1.03	0.2712
	Single Mean	-58.07	0.0012	-5.34	<.0001
	Trend	-120.13	0.0001	-7.47	<.0001
CA	Zero Mean	-1.03	0.4661	-0.67	0.4245
	Single Mean	-57.02	0.0012	-5.30	<.0001
	Trend	-57.42	0.0005	-5.30	0.0001
EM	Zero Mean	-0.91	0.4877	-0.66	0.4316
	Single Mean	-101.82	0.0001	-7.04	<.0001
	Trend	-139.79	0.0001	-8.21	<.0001
AC	Zero Mean	-0.51	0.5666	-0.36	0.5531
	Single Mean	-25.35	0.0024	-3.44	0.0113
	Trend	-58.80	0.0005	-5.36	0.0001
OEE	Zero Mean	-0.59	0.5488	-0.44	0.5231
	Single Mean	-44.56	0.0012	-4.52	0.0003
	Trend	-54.84	0.0005	-5.11	0.0002

4.4.3 Sample cross correlations

Table 4.59 and table A1.3 (Appendix A) have the correlation of 0.82 at lag 0 between EM and LQ and thus shows that there is a strong positive relationship between them. The values less than close or less than 0.4 will be indicating that there is a little or no correlation between the variables.

Table 4.59 Schematic Representation of Cross Correlations

Schematic Representation of Cross Correlations											
Variable/Lag	0	1	2	3	4	5	6	7	8	9	10
LQ	+++++	+++++	+.+++	+.+++	+++++	+.+++	+.+++	+.+++	+.+++	+.+++	+.+++
CA	+++++	++.+.	.+...	.+...	.+..+	.+.++	.+...	.+...	+++++	.++.+	.+...
EM	+++++	+++++	+.+++	+.+++	+.+++	+.+++	+.+.+	+.+.+	+.+++	...++	.....
AC	+++++	+++++	+++++	+++++	+++++	+++++	+++++	+++++	+++++	+++++	+++++
OEE	+++++	+.+++	+.+++	+.+++	+.+++	+.+++	+.+.+	+.+.+	+.+++	..+.+	.-..+
+ is > 2*std error, - is < -2*std error, . is between											

4.4.4 Tentative order selection

Table 4.60 and table A1.4 (Appendix A) suggest that VAR (3) is appropriate. Minimum information criterion method was used in fitting several models with different values of *p* and *q* and then selecting the model with the minimum value of the information. According to minic (minimum information criterion) method, the smallest value of the criterion is 24.46.

Table 4.60 Minimum information based on AICC

Minimum Information Criterion Based on AICC					
Lag	MA 0	MA 1	MA 2	MA 3	MA 4
AR 0	26.797796	26.665288	26.767769	26.734155	26.807283
AR 1	24.897163	24.557788	24.758209	24.757078	24.848466
AR 2	24.665451	24.556510	24.655061	24.786243	25.020200
AR 3	24.455190	24.458712	24.705670	24.993883	25.256593
AR 4	24.616783	24.649956	24.984307	25.226616	25.491020

Table 4.61 is a schematic representation of partial autoregression. The table works as a guideline to determine the order of the VAR model by constructing a confidence interval of ± 2 estimated standard errors. Each element of the partial autoregression matrix is classified as a "-", "." or "+" depending on whether it is below confidence limit, between confidence limits or above confidence limit.

Table 4.61 Schematic representation of Partial Autoregression

Schematic Representation of Partial Autoregression										
Variable/Lag	1	2	3	4	5	6	7	8	9	10
LQ	+....	...+.	+.-+.	.....	.....	.....	.....	.+...	.....	.....
CA	.+...	...+.	.....	.....	.....	.....	.....	.....	..-+.	.....
EM	.....	..-+.	.....	.....	.....	.....	.+...	.+...	.....	.....
AC	+.-+.	...+.	..-+.	.....	.....	.....	.....	....+	.....	.....
OEE	....+	..-.+	.....	.....	...-.	.....	.....	.....	.....	.....
+ is > 2*std error, - is < -2*std error, . is between										

4.4.5 Model estimation

Multivariate time series analysis extends many of the ideas of univariate time series analysis to systems of equations. The primary model in time series analysis is the vector autoregression (VAR), a direct and natural extension of the univariate autoregression. The VAR process is the mechanism that is used to link multiple stationary time series variable together.

Table 4.62 and table A1.5 (Appendix A) provide the method and model for the five variables of price indices of electrical appliances. A VAR (3) was fitted using the method of the least squares.

Table 4.62 Type of model selected

Type of Model	VAR(3)
Estimation Method	Least Squares Estimation

4.4.6 Multivariate diagnostics

Diagnostics check is performed in order to determine whether the selected model is an adequate model representation of the data, and if the model is adequate the residuals should have no significant trend or pattern.

Table 4.63 shows different information criteria methods that were used in the data but the one which was considered is AICC method with 24.46.

Table 4.63 Information Criteria

Information Criteria	
AICC	24.45519
HQC	24.97673
AIC	24.33306
SBC	25.91760
FPEC	3.71E10

4.4.7 Forecasts

Forecasted values using multivariate time series are provided in table A1.6 (Appendix A) for the five variables for the period of one year.

CHAPTER 5: SUMMARY, CONCLUSION AND RECOMMENDATIONS

This chapter focuses on the discussion of findings, conclusion and recommendations.

In this study price indices of electrical appliances in South Africa using Stats SA's "Manufacturing: Production and Sales" data were analysed. The data was monthly collected from January 1998 to December 2010 (2005 was used as a base year). The ultimate aim of this study was to analyse the price indices of electrical appliances in South Africa in order to determine the factors affecting the price of electrical appliances. The impact of price increases on disposable income of consumers was also determined. The final step was to predict price indices of electrical appliances by building statistical models for forecasting.

5.1 Summary and conclusion

Univariate (ARIMA and Exponential smoothing) time series, multivariate time series, regression and neural networks were applied to price indices of the five electrical appliances, namely :(lighting equipment, electric machines, other electrical equipment, communication apparatus and accumulators).

Regression analysis was applied to the lighting equipment variable to check for a monthly effect after its plot depicted some seasonality pattern. Only February did not have an impact or an effect on time since it was found not to be significantly different from zero. December was used as the reference variable.

Based on the monthly effect the impact of price increases on disposable income of consumers was largely felt because of the high cost of electrical appliances such as heaters (electric machines) during winter season.

ARIMA models were fitted to three variables namely: lighting equipment, electric machines and other electric machines. The other two variables considered in this research, accumulators and communication apparatus, failed the diagnostic test for ARIMA. ARCH models were then applied to the two variables: accumulators and communication apparatus that failed some of the tests when fitting ARIMA models. ARCH managed to give the model which satisfied the diagnostic of the residuals.

The best models from exponential smoothing, ARIMA models and neural networks were then employed in the study for forecasting. The first comparison was made among the three types of exponential smoothing (single, double and triple exponential smoothing). Triple exponential smoothing was found to be the best model for forecasting compared to SES and DES based on the accuracy measures. Comparing the accuracy measures of the three techniques (ARIMA, ES and Neural networks), neural networks outperformed the exponential smoothing model and ARIMA.

Multivariate time series was also done with respect to the five variables. The descriptive statistics of the price indices of electrical appliances were also performed.

Then using the Dickey-Fuller unit root test all variables were stationary except accumulators with the single mean of 0.0113. A sample cross correlation was done and it revealed a strong positive correlation between electrical machines and lighting equipment (0.82) at lag 0 and the lowest correlated variables were between LQ and CA of about 0.37 at lag 0.

The VAR (3) model was selected based on the smallest AICC (24.46). The forecasted values using VAR (3) model for the five variables considered under the research has been displayed in table A1.6 (Appendix A) for one year.

Neural networks were then used for the prediction of forecast.

5.2 Recommendations

Forecasting is a very important key for the consumers of electrical appliances. During summer season electrical appliances such as electric fans and air conditioning plants are used to provide us with cool atmosphere. In winter, heaters and air conditioners are used to keep the rooms warm. Also refrigerators are very important for food and beverage preservations. Communication apparatus are in much use in the modern era. Most companies and households will not function in the absence of these speedy means of communication.

The microwaves and kettles in the majority of workplaces and households get used at regular times. Millions of newspapers, magazines and books are published daily. These days everyone can find a book of his/her choice and of a relevant standard with the help of electrical appliances. Life itself will not be worth living in the absence of such means of invention.

The government has to take note of forecasting the price indices for electrical appliances so that the consumers can have time in planning ahead. Awareness can also play an important role because it will be alerting consumers of the right time to purchase these electrical appliances. It is also important to take into consideration the quality and price of the appliances to avoid buying them more often as there is a relationship between price and quality of appliances.

With escalating prices of electricity from time to time, information communicated to consumers could assist them to more cost-effective electrical appliances.

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APPENDIX A

A1 RESULTS OF MULTIVARIATE TIME SERIES

Table A1. 1 Simple Summary Statistics

Variable	Type	N	Mean	Standard Deviation	Min	Max	Label
LQ	Dependent	156	107.85128	26.07808	53.10	184.50	LIGHTING EQUIPMENT
CA	Dependent	156	98.83910	17.57215	53.80	136.50	COMMUNICATION APPARATUS
EM	Dependent	156	110.61346	14.24103	74.70	145.10	ELECTRIC MACHINES
AC	Dependent	156	102.28333	22.89844	55.40	168.00	ACCUMMULATORS
OEE	Dependent	156	130.15769	22.31377	76.30	192.20	OTHER ELECTRICAL EQUIPMENT

Table A1. 2 Cross Covariances of Dependent Series

Lag	Variable	LQ	CA	EM	AC	OEE
0	LQ	675.70699	169.48062	302.73751	437.22419	309.27749
	CA	169.48062	306.80110	96.75723	155.34001	106.65967
	EM	302.73751	96.75723	201.50693	211.02881	252.27249
	AC	437.22419	155.34001	211.02881	520.97729	210.46506
	OEE	309.27749	106.65967	252.27249	210.46506	494.71270
1	LQ	423.15925	118.56756	182.42429	365.65653	226.10143
	CA	108.61587	172.60125	32.20097	95.04785	39.11621
	EM	196.39607	43.46931	105.84505	142.60563	152.92846
	AC	328.14336	107.42905	128.40864	366.38498	155.64947
	OEE	229.48723	36.05909	147.28415	153.82627	300.61897
2	LQ	267.61588	21.98880	63.55264	243.84504	138.30757
	CA	60.90956	113.52307	-6.35419	53.58328	-6.12156
	EM	111.52406	-14.17520	33.04663	82.25022	78.70814
	AC	283.99078	85.56319	89.19071	320.75281	124.40576
	OEE	138.97746	-34.99041	81.71607	98.05289	216.17290
3	LQ	291.27074	50.46072	89.27625	258.20522	173.38152
	CA	32.65144	115.64443	-0.12873	32.17615	38.33784
	EM	98.52512	17.05068	47.04563	75.80462	115.68355
	AC	295.76188	125.14256	116.49529	335.66285	161.06550
	OEE	128.52433	1.59769	94.53789	81.67043	253.95148
4	LQ	256.10762	89.53525	125.14770	269.11842	237.37939
	CA	41.28786	119.56115	21.58931	61.58615	78.05846
	EM	102.01413	36.66117	68.21501	84.32986	156.76769
	AC	283.92478	122.02132	119.21797	303.48801	180.90271
	OEE	160.63850	17.25518	124.69946	100.02065	293.33666
5	LQ	224.41703	15.64287	94.34868	221.19014	216.00302
	CA	52.44786	103.48229	28.73684	68.86420	70.36117
	EM	93.24300	1.69796	52.11989	76.12366	126.76047
	AC	249.26692	94.58017	84.60075	285.16331	130.56492
	OEE	150.46363	-0.96172	110.69168	95.50866	256.63094
6	LQ	248.28776	-12.64908	92.51903	215.38575	166.83860
	CA	6.83203	71.33637	6.34586	31.11497	49.18980
	EM	75.47817	-22.57847	39.73269	43.90710	98.00549
	AC	219.14199	77.80095	65.48095	238.87518	104.65510
	OEE	116.19350	-29.00883	87.74490	45.38573	210.98666
7	LQ	205.51492	13.06011	89.45055	198.59544	163.46111
	CA	41.19424	69.63514	29.32516	36.86362	61.98868
	EM	71.21431	-7.24041	43.24217	45.47139	106.21305
	AC	230.55567	96.31483	80.33795	243.50241	127.98692
	OEE	146.51244	-25.59802	99.76308	39.30028	218.14650
8	LQ	235.42317	11.24564	95.62729	221.85662	176.31016
	CA	118.16149	95.25049	78.15605	74.61309	114.32304
	EM	96.61380	-6.75494	56.57560	72.81942	115.35395
	AC	254.02040	103.87450	87.68922	252.64155	138.47549
	OEE	170.04519	-5.79058	131.07962	97.39703	235.55669
9	LQ	233.12059	-13.05074	82.71257	229.68029	133.06061
	CA	71.49550	82.99268	55.97102	63.58092	104.44720
	EM	57.99910	-35.02720	29.46015	60.95007	76.05468
	AC	230.72132	87.69953	72.65614	247.56757	104.18518
	OEE	89.54563	-41.92379	76.93225	59.81911	184.44693
10	LQ	199.34247	21.92255	80.42569	207.49859	114.67047
	CA	21.16158	61.73776	7.94092	18.48219	43.03833
	EM	20.51043	-36.74252	6.74199	26.08434	40.84862
	AC	197.46870	92.53515	60.17003	216.32743	85.66982
	OEE	35.91491	-75.02277	21.87441	-8.68471	105.98937

Table A1. 3 Cross Correlations of Dependent Series

Lag	Variable	LQ	CA	EM	AC	OEE
	LQ	1.00000	0.37223	0.82043	0.73691	0.53492
	CA	0.37223	1.00000	0.38914	0.38855	0.27378
	EM	0.82043	0.38914	1.00000	0.65131	0.79900
	AC	0.73691	0.38855	0.65131	1.00000	0.41457
0	OEE	0.53492	0.27378	0.79900	0.41457	1.00000
	LQ	0.62625	0.26041	0.49438	0.61629	0.39106
	CA	0.23855	0.56258	0.12951	0.23774	0.10040
	EM	0.53224	0.17483	0.52527	0.44013	0.48436
	AC	0.55306	0.26871	0.39631	0.70326	0.30659
1	OEE	0.39692	0.09256	0.46648	0.30300	0.60766
	LQ	0.39605	0.04829	0.17223	0.41098	0.23922
	CA	0.13378	0.37002	-0.02556	0.13403	-0.01571
	EM	0.30223	-0.05701	0.16400	0.25385	0.24929
	AC	0.47865	0.21402	0.27527	0.61568	0.24505
2	OEE	0.24037	-0.08981	0.25881	0.19314	0.43697
	LQ	0.43106	0.11083	0.24194	0.43519	0.29988
	CA	0.07171	0.37694	-0.00052	0.08048	0.09841
	EM	0.26701	0.06858	0.23347	0.23396	0.36640
	AC	0.49849	0.31302	0.35955	0.64429	0.31726
3	OEE	0.22229	0.00410	0.29942	0.16087	0.51333
	LQ	0.37902	0.19665	0.33916	0.45358	0.41057
	CA	0.09068	0.38970	0.08683	0.15404	0.20036
	EM	0.27646	0.14745	0.33852	0.26027	0.49652
	AC	0.47854	0.30521	0.36795	0.58254	0.35634
4	OEE	0.27784	0.04429	0.39495	0.19702	0.59294
	LQ	0.33212	0.03436	0.25569	0.37280	0.37360
	CA	0.11519	0.33729	0.11558	0.17225	0.18060
	EM	0.25269	0.00683	0.25865	0.23494	0.40148
	AC	0.42012	0.23657	0.26111	0.54736	0.25718
5	OEE	0.26024	-0.00247	0.35059	0.18813	0.51875
	LQ	0.36745	-0.02778	0.25073	0.36302	0.28856
	CA	0.01501	0.23252	0.02552	0.07783	0.12626
	EM	0.20455	-0.09081	0.19718	0.13551	0.31041
	AC	0.36935	0.19460	0.20210	0.45851	0.20615
6	OEE	0.20097	-0.07446	0.27791	0.08940	0.42648
	LQ	0.30415	0.02868	0.24241	0.33472	0.28272
	CA	0.09048	0.22697	0.11794	0.09221	0.15911
	EM	0.19299	-0.02912	0.21459	0.14034	0.33640
	AC	0.38859	0.24091	0.24795	0.46740	0.25210
7	OEE	0.25341	-0.06571	0.31597	0.07741	0.44096
	LQ	0.34841	0.02470	0.25915	0.37392	0.30495
	CA	0.25952	0.31046	0.31433	0.18663	0.29345
	EM	0.26183	-0.02717	0.28076	0.22475	0.36535
	AC	0.42813	0.25982	0.27064	0.48494	0.27276
8	OEE	0.29411	-0.01486	0.41516	0.19185	0.47615
	LQ	0.34500	-0.02866	0.22415	0.38711	0.23014
	CA	0.15703	0.27051	0.22511	0.15903	0.26810
	EM	0.15718	-0.14087	0.14620	0.18811	0.24088
	AC	0.38887	0.21936	0.22424	0.47520	0.20522
9	OEE	0.15488	-0.10761	0.24366	0.11783	0.37284
	LQ	0.29501	0.04815	0.21796	0.34972	0.19833
	CA	0.04648	0.20123	0.03194	0.04623	0.11047
	EM	0.05558	-0.14777	0.03346	0.08051	0.12938
	AC	0.33282	0.23146	0.18571	0.41523	0.16875
10	OEE	0.06212	-0.19257	0.06928	-0.01711	0.21424

Table A1. 4 Partial Autoregression

Lag	Variable	LQ	CA	EM	AC	OEE
1	LQ	0.50722	-0.03251	-0.24643	0.24248	0.17630
	CA	0.16204	0.55971	-0.36650	0.04408	0.01905
	EM	0.13564	-0.08554	0.13752	0.04505	0.14207
	AC	0.43109	-0.05023	-0.79732	0.59658	0.20505
	OEE	0.18330	-0.12230	-0.37983	0.05695	0.68890
2	LQ	-0.02531	-0.02659	-0.46948	0.36797	0.05279
	CA	-0.13916	0.15837	-0.32556	0.20207	-0.04715
	EM	-0.07473	-0.04741	-0.50358	0.20959	0.18915
	AC	-0.10798	-0.08803	-0.51810	0.44733	0.09504
	OEE	0.14012	-0.10113	-1.03999	0.22347	0.49170
3	LQ	0.48951	-0.16610	-0.85389	0.29945	0.29161
	CA	-0.10720	0.07070	0.22210	0.14358	0.04129
	EM	0.05514	-0.07308	-0.06123	0.14757	0.11906
	AC	0.18007	-0.15872	-0.63561	0.40868	0.16644
	OEE	-0.00743	0.05668	0.13878	0.02532	0.22593
4	LQ	-0.07514	-0.01763	-0.33311	0.11958	0.24423
	CA	-0.07890	0.15136	0.24512	-0.08547	-0.05941
	EM	-0.03278	0.06549	-0.04055	-0.00628	0.12812
	AC	-0.08464	0.17837	-0.00873	-0.05202	0.01931
	OEE	-0.07281	0.16123	0.23048	-0.10272	0.15634
5	LQ	-0.11000	0.11553	0.05496	-0.11337	0.03277
	CA	-0.17045	0.17357	-0.12504	0.04781	0.08387
	EM	-0.04725	0.13391	0.07352	-0.15016	0.00694
	AC	-0.23243	0.14459	0.18990	0.02044	-0.06068
	OEE	0.11899	0.15106	0.06395	-0.29425	0.06099
6	LQ	0.23286	-0.21938	-0.05178	-0.10563	0.05714
	CA	-0.04198	-0.02448	-0.03644	0.02592	0.06766
	EM	0.10481	-0.08484	0.06742	-0.13790	0.04446
	AC	0.11041	-0.05780	-0.13381	-0.08309	-0.00247
	OEE	0.07427	-0.02093	0.26831	-0.24905	-0.03348
7	LQ	-0.02486	0.21944	-0.63881	0.17823	0.35013
	CA	-0.13320	0.07726	0.37318	-0.04244	-0.09928
	EM	-0.05821	0.17139	-0.18162	0.01261	0.13807
	AC	-0.18304	0.07985	0.27229	0.05318	-0.12869
	OEE	-0.14619	0.08841	0.11520	-0.03126	0.07656
8	LQ	0.08341	0.31676	-0.33541	0.04432	0.09549
	CA	0.03601	0.19976	-0.50993	0.08576	0.19231
	EM	0.01609	0.20680	-0.19095	-0.06302	0.17941
	AC	0.10529	0.11837	-0.40882	-0.01164	0.30729
	OEE	-0.00389	0.20466	-0.02700	-0.04844	0.08730
9	LQ	0.02147	-0.15272	-0.28763	0.20539	-0.14554
	CA	0.02717	0.02402	-0.78516	0.26156	0.21728
	EM	0.03906	0.00508	-0.35370	0.07838	0.03865
	AC	0.07351	-0.06319	-0.34439	0.16097	0.08414
	OEE	0.03405	0.16435	-0.40760	-0.00242	0.06960
10	LQ	0.29001	-0.15376	-0.32874	-0.05989	-0.04047
	CA	0.16485	0.05095	-0.18074	0.05413	-0.07431
	EM	0.17211	-0.10808	-0.10459	-0.04039	-0.10339
	AC	0.23604	0.00925	-0.23363	-0.09931	-0.06872
	OEE	0.13532	-0.14031	-0.17131	0.00520	-0.13323

Table A1. 5 Model Parameter Estimates

Equation	Parameter	Estimate	Standard Error	t Value	Pr > |t|	Variable
LQ	CONST1	34.39722	23.83365	1.44	0.1512	1
	AR1_1_1	0.32793	0.14127	2.32	0.0217	LQ(t-1)
	AR1_1_2	0.07004	0.12862	0.54	0.5869	CA(t-1)
	AR1_1_3	0.72369	0.37734	1.92	0.0572	EM(t-1)
	AR1_1_4	-0.12582	0.14266	-0.88	0.3793	AC(t-1)
	AR1_1_5	-0.18414	0.16587	-1.11	0.2689	OEE(t-1)
	AR2_1_1	-0.22027	0.14548	-1.51	0.1323	LQ(t-2)
	AR2_1_2	0.00134	0.13678	0.01	0.9922	CA(t-2)
	AR2_1_3	-0.13887	0.37940	-0.37	0.7149	EM(t-2)
	AR2_1_4	0.13085	0.14701	0.89	0.3750	AC(t-2)
	AR2_1_5	-0.03616	0.16767	-0.22	0.8296	OEE(t-2)
	AR3_1_1	0.51198	0.14306	3.58	0.0005	LQ(t-3)
	AR3_1_2	-0.16921	0.12510	-1.35	0.1784	CA(t-3)
	AR3_1_3	-0.89250	0.35966	-2.48	0.0143	EM(t-3)
	AR3_1_4	0.33463	0.14256	2.35	0.0203	AC(t-3)
	AR3_1_5	0.34386	0.16515	2.08	0.0392	OEE(t-3)
CA	CONST2	33.40171	16.51110	2.02	0.0450	1
	AR1_2_1	0.16810	0.09787	1.72	0.0881	LQ(t-1)
	AR1_2_2	0.48153	0.08910	5.40	0.0001	CA(t-1)
	AR1_2_3	0.03387	0.26141	0.13	0.8971	EM(t-1)
	AR1_2_4	-0.07990	0.09883	-0.81	0.4202	AC(t-1)
	AR1_2_5	-0.01319	0.11491	-0.11	0.9088	OEE(t-1)
	AR2_2_1	-0.21908	0.10078	-2.17	0.0314	LQ(t-2)
	AR2_2_2	0.10583	0.09476	1.12	0.2660	CA(t-2)
	AR2_2_3	-0.28099	0.26284	-1.07	0.2869	EM(t-2)
	AR2_2_4	0.16782	0.10184	1.65	0.1017	AC(t-2)
	AR2_2_5	-0.10433	0.11615	-0.90	0.3706	OEE(t-2)
	AR3_2_1	-0.08521	0.09910	-0.86	0.3914	LQ(t-3)
	AR3_2_2	0.07479	0.08666	0.86	0.3897	CA(t-3)
	AR3_2_3	0.18845	0.24916	0.76	0.4507	EM(t-3)
	AR3_2_4	0.16884	0.09876	1.71	0.0896	AC(t-3)
	AR3_2_5	0.08076	0.11441	0.71	0.4814	OEE(t-3)
EM	CONST3	58.13641	13.53351	4.30	0.0001	1
	AR1_3_1	0.10120	0.08022	1.26	0.2093	LQ(t-1)
	AR1_3_2	0.01522	0.07304	0.21	0.8352	CA(t-1)
	AR1_3_3	0.64931	0.21427	3.03	0.0029	EM(t-1)
	AR1_3_4	-0.12305	0.08101	-1.52	0.1311	AC(t-1)
	AR1_3_5	-0.08247	0.09419	-0.88	0.3828	OEE(t-1)
	AR2_3_1	-0.17109	0.08261	-2.07	0.0402	LQ(t-2)
	AR2_3_2	-0.04430	0.07767	-0.57	0.5694	CA(t-2)
	AR2_3_3	-0.46025	0.21544	-2.14	0.0344	EM(t-2)
	AR2_3_4	0.15061	0.08348	1.80	0.0734	AC(t-2)
	AR2_3_5	0.14855	0.09521	1.56	0.1210	OEE(t-2)
	AR3_3_1	0.08062	0.08123	0.99	0.3227	LQ(t-3)
	AR3_3_2	-0.07260	0.07104	-1.02	0.3086	CA(t-3)
	AR3_3_3	-0.08894	0.20422	-0.44	0.6639	EM(t-3)
	AR3_3_4	0.17655	0.08095	2.18	0.0309	AC(t-3)
	AR3_3_5	0.16315	0.09378	1.74	0.0842	OEE(t-3)
AC	CONST4	77.26063	16.99515	4.55	0.0001	1
	AR1_4_1	0.34857	0.10074	3.46	0.0007	LQ(t-1)
	AR1_4_2	0.01734	0.09172	0.19	0.8503	CA(t-1)
	AR1_4_3	-0.14230	0.26907	-0.53	0.5978	EM(t-1)
	AR1_4_4	0.19908	0.10173	1.96	0.0524	AC(t-1)
	AR1_4_5	-0.02881	0.11828	-0.24	0.8079	OEE(t-1)
	AR2_4_1	-0.17467	0.10374	-1.68	0.0945	LQ(t-2)
	AR2_4_2	-0.02540	0.09754	-0.26	0.7949	CA(t-2)
	AR2_4_3	-0.27770	0.27054	-1.03	0.3065	EM(t-2)
	AR2_4_4	0.25566	0.10483	2.44	0.0160	AC(t-2)
	AR2_4_5	0.05986	0.11956	0.50	0.6174	OEE(t-2)
	AR3_4_1	0.18816	0.10201	1.84	0.0673	LQ(t-3)
	AR3_4_2	-0.17527	0.08921	-1.96	0.0515	CA(t-3)
	AR3_4_3	-0.69888	0.25646	-2.73	0.0073	EM(t-3)
	AR3_4_4	0.45867	0.10166	4.51	0.0001	AC(t-3)
	AR3_4_5	0.23740	0.11776	2.02	0.0458	OEE(t-3)
OEE	CONST5	36.24153	19.65961	1.84	0.0674	1
	AR1_5_1	0.02663	0.11653	0.23	0.8196	LQ(t-1)
	AR1_5_2	0.07475	0.10610	0.70	0.4823	CA(t-1)
	AR1_5_3	0.46374	0.31125	1.49	0.1385	EM(t-1)
	AR1_5_4	-0.12618	0.11768	-1.07	0.2855	AC(t-1)
	AR1_5_5	0.25270	0.13682	1.85	0.0669	OEE(t-1)
	AR2_5_1	-0.03043	0.12000	-0.25	0.8002	LQ(t-2)
	AR2_5_2	-0.17599	0.11283	-1.56	0.1211	CA(t-2)
	AR2_5_3	-0.98984	0.31296	-3.16	0.0019	EM(t-2)
	AR2_5_4	0.24620	0.12127	2.03	0.0443	AC(t-2)
	AR2_5_5	0.40293	0.13830	2.91	0.0042	OEE(t-2)
	AR3_5_1	0.03072	0.11800	0.26	0.7950	LQ(t-3)
	AR3_5_2	0.04636	0.10319	0.45	0.6540	CA(t-3)
	AR3_5_3	0.12242	0.29667	0.41	0.6805	EM(t-3)
	AR3_5_4	0.06400	0.11759	0.54	0.5871	AC(t-3)
	AR3_5_5	0.28939	0.13623	2.12	0.0354	OEE(t-3)

Table A1. 6 Forecasts

Variable	Obs	Forecast	Standard Error	95% Confidence Limits	
LQ	157	103.82648	18.67673	67.22077	140.43220
	158	135.10234	21.63721	92.69418	177.51050
	159	135.10576	21.83135	92.31710	177.89442
	160	132.35126	22.57571	88.10367	176.59885
	161	127.84389	23.10495	82.55901	173.12876
	162	126.62595	23.43864	80.68705	172.56484
	163	130.34132	23.86800	83.56090	177.12173
	164	127.69958	24.17746	80.31263	175.08654
	165	124.13445	24.46087	76.19203	172.07688
	166	124.90920	24.80979	76.28291	173.53549
	167	124.39387	25.07073	75.25614	173.53159
	168	122.95097	25.30451	73.35503	172.54690
CA	157	86.51797	12.93857	61.15885	111.87710
	158	116.07019	14.92668	86.81443	145.32596
	159	118.61271	15.79018	87.66454	149.56089
	160	101.75406	16.65078	69.11912	134.38900
	161	105.45309	17.08120	71.97456	138.93162
	162	110.40802	17.31248	76.47619	144.33986
	163	106.42927	17.56205	72.00828	140.85025
	164	104.40048	17.74573	69.61949	139.18147
	165	103.94694	17.86810	68.92612	138.96776
	166	102.95575	17.95803	67.75865	138.15284
	167	102.49948	18.04632	67.12934	137.86963
	168	101.71263	18.12134	66.19546	137.22980
EM	157	107.38086	10.60524	86.59498	128.16675
	158	137.72875	12.63635	112.96197	162.49554
	159	138.90971	12.76542	113.88995	163.92946
	160	125.45760	13.26197	99.46461	151.45059
	161	125.11731	13.65827	98.34760	151.88703
	162	127.04832	13.80376	99.99345	154.10318
	163	125.59209	13.91014	98.32872	152.85547
	164	124.14142	14.07411	96.55666	151.72617
	165	122.61662	14.20272	94.77981	150.45344
	166	122.05349	14.31631	93.99404	150.11293
	167	122.10189	14.42802	93.82348	150.38030
	168	121.33065	14.52108	92.86986	149.79144
AC	157	113.86360	13.31788	87.76103	139.96616
	158	136.40091	14.98576	107.02937	165.77245
	159	136.72940	15.28678	106.76786	166.69094
	160	127.23349	16.71689	94.46898	159.99800
	161	123.46743	17.70822	88.75997	158.17490
	162	120.70007	18.25395	84.92299	156.47715
	163	121.59867	18.90234	84.55077	158.64657
	164	120.23382	19.48628	82.04141	158.42623
	165	116.77065	19.95130	77.66682	155.87447
	166	116.73547	20.46367	76.62740	156.84353
	167	116.83717	20.89864	75.87659	157.79776
	168	115.55664	21.25808	73.89156	157.22171
OEE	157	151.76812	15.40583	121.57325	181.96298
	158	182.82548	17.48486	148.55578	217.09518
	159	181.69966	18.03519	146.35134	217.04798
	160	159.72135	19.22842	122.03434	197.40836
	161	162.28042	20.31700	122.45983	202.10101
	162	168.53131	20.79012	127.78342	209.27919
	163	162.13956	21.17073	120.64569	203.63343
	164	159.11905	21.62427	116.73627	201.50184
	165	159.36807	21.99500	116.25866	202.47748
	166	157.31157	22.26130	113.68022	200.94292
	167	156.11279	22.52984	111.95511	200.27047
	168	155.17581	22.77553	110.53660	199.81503

A2 EXPONENTIAL SMOOTHING RESULTS USING SINGLE AND DOUBLE EXPONENTIAL SMOOTHING

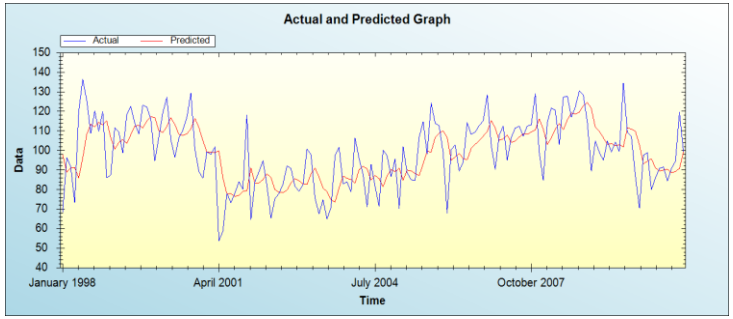
A2.1 Analysis for communication apparatus using SES

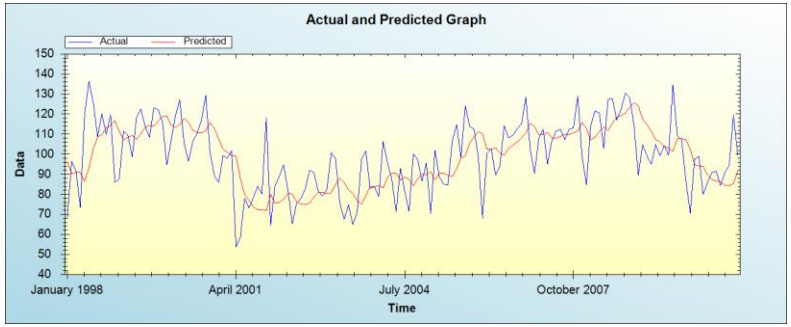
Single Exponential Smoothing	
Variable	Communication apparatus
Smoothing Constant	
Alpha (for data)	0.3
Accuracy Measures	
Mean Absolute Error (MAE)	11.05716
Sum Square Error (SSE)	33225.93
Mean Squared Error (MSE)	212.9867
Mean Percentage Error (MPE)	-2.07491
Mean Absolute Percentage Error (MAPE)	11.90639

Analysis for communication apparatus using DES

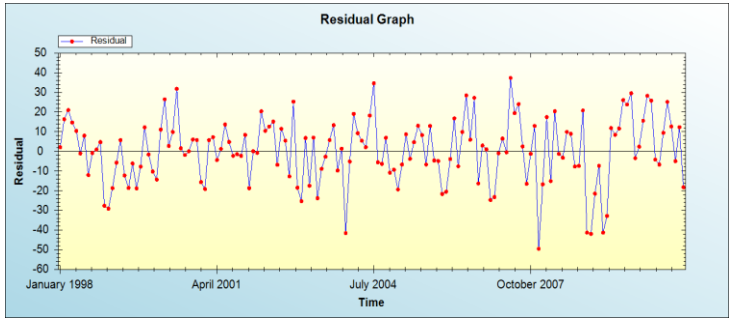
Double Exponential Smoothing	
Variable	Communication apparatus
Smoothing Constant	
Alpha (for data)	0.2
Gamma (for trend)	0.1
Accuracy Measures	
Mean Absolute Error (MAE)	11.31952
Sum Square Error (SSE)	35558.06
Mean Squared Error (MSE)	227.9363
Mean Percentage Error (MPE)	-2.03383
Mean Absolute Percentage Error (MAPE)	12.20871

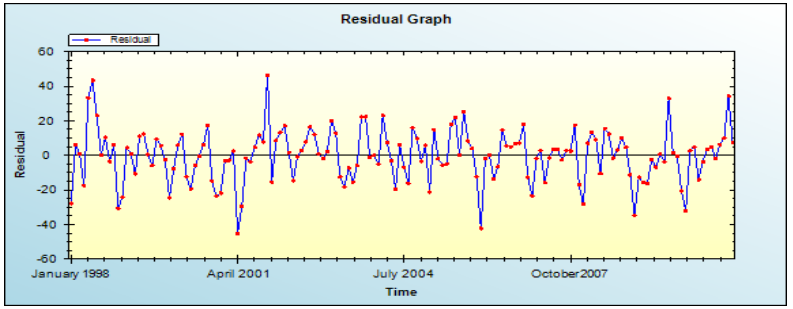
Time series plot of communication apparatus against the predicted using SES and DES





Residual analysis of communication apparatus using SES and DES



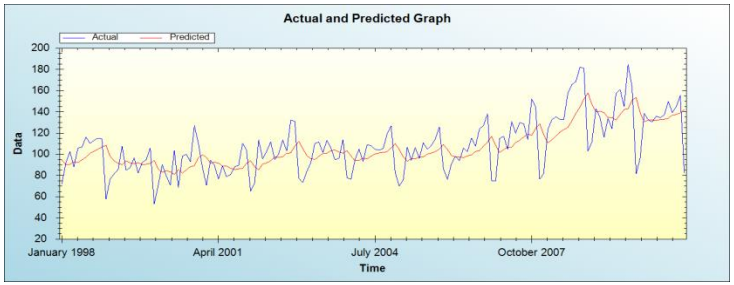


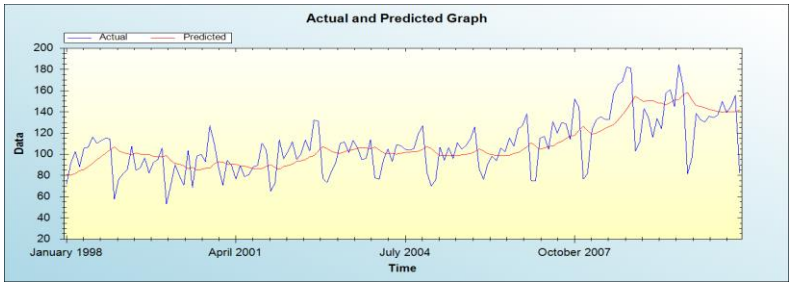
A2.2 Analysis for lighting equipment using SES and DES

SES	
Variable	Lighting equipments
Smoothing Constant	
Alpha (for data)	0.2
Accuracy Measures	
Mean Absolute Error (MAE)	15.62779
Sum Square Error (SSE)	65044.05
Mean Squared Error (MSE)	416.9491
Mean Percentage Error (MPE)	-2.55327
Mean Absolute Percentage Error (MAPE)	15.8997

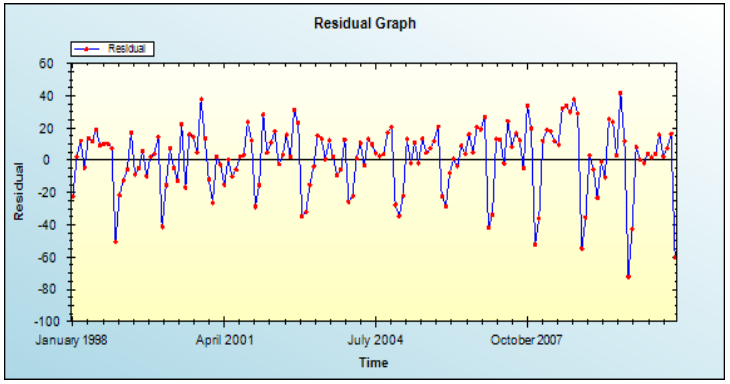
DES	
Variable	Lighting equipments
Smoothing Constant	
Alpha (for data)	0.1
Gamma (for trend)	0.1
Accuracy Measures	
Mean Absolute Error (MAE)	16.46535
Sum Square Error (SSE)	69671.5
Mean Squared Error (MSE)	446.6122
Mean Percentage Error (MPE)	-4.11169
Mean Absolute Percentage Error (MAPE)	16.79785

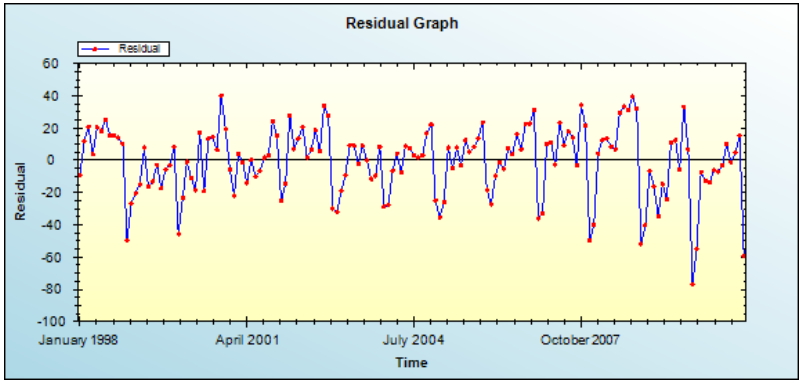
Time series plot using SES and DES of lighting equipment





Reiduals using SES and DES of lighting equipment



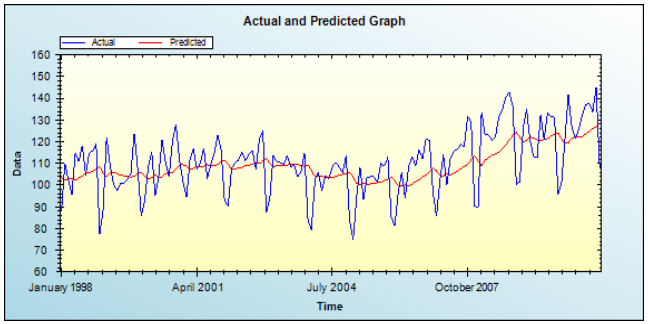


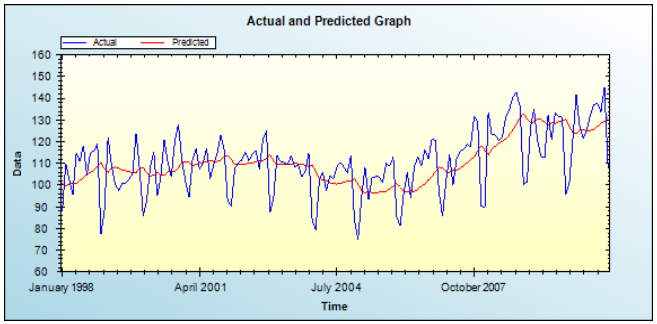
A2.3 Analysis for electric machines using and DES

SES	
Variable	Electric machines
Smoothing Constant	
Alpha (for data)	0.1
Accuracy Measures	
Mean Absolute Error (MAE)	9.812919
Sum Square Error (SSE)	23623.91
Mean Squared Error (MSE)	151.4354
Mean Percentage Error (MPE)	0.011204
Mean Absolute Percentage Error (MAPE)	9.214591

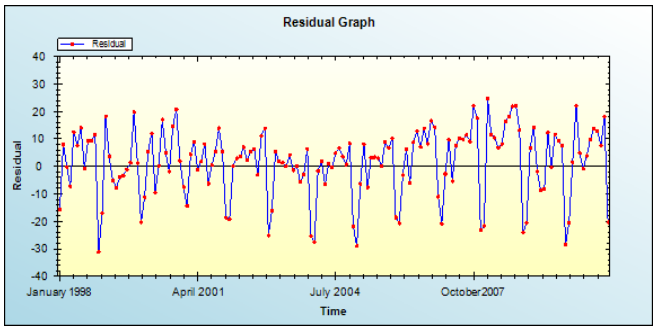
DES	
Variable	Electric machines
Smoothing Constant	
Alpha (for data)	0.1
Gamma (for trend)	0.1
Accuracy Measures	
Mean Absolute Error (MAE)	9.660573
Sum Square Error (SSE)	24015.37
Mean Squared Error (MSE)	153.9447
Mean Percentage Error (MPE)	-1.1232
Mean Absolute Percentage Error (MAPE)	9.238326

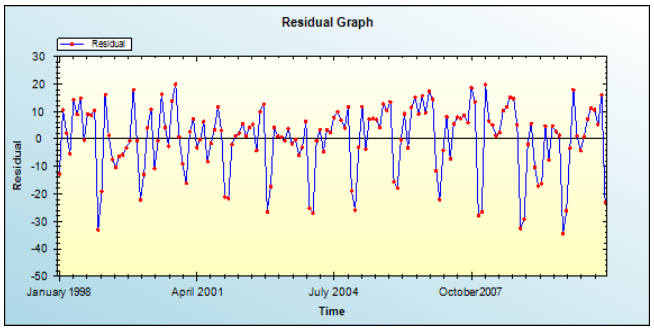
Time series plot using SES and DES of electric machines





Residual analysis for SES and DES of electric machines



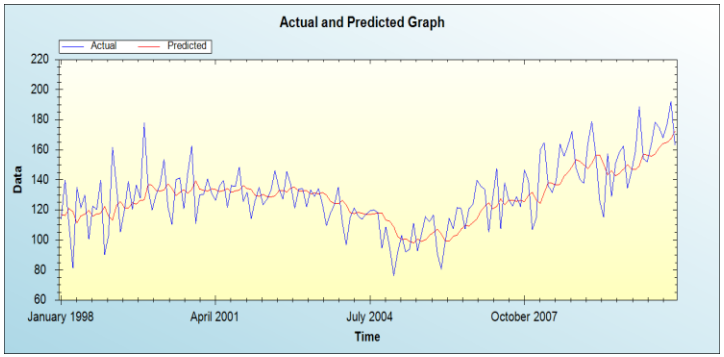


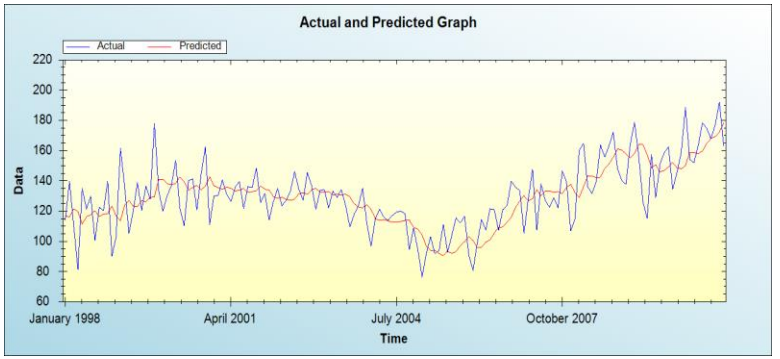
A2.4 Analysis for other electrical equipment using SES and DES

SES	
Variable	Other electrical equipment
Smoothing Constant	
Alpha (for data)	0.2
Accuracy Measures	
Mean Absolute Error (MAE)	12.35709
Sum Square Error (SSE)	39575.35
Mean Squared Error (MSE)	253.6881
Mean Percentage Error (MPE)	-0.20997
Mean Absolute Percentage Error (MAPE)	9.74933

DES	
Variable	Other electrical equipment
Smoothing Constant	
Alpha (for data)	0.2
Gamma (for trend)	0.1
Accuracy Measures	
Mean Absolute Error (MAE)	12.47085
Sum Square Error (SSE)	39767.38645
Mean Squared Error (MSE)	254.919144
Mean Percentage Error (MPE)	-0.908801
Mean Absolute Percentage Error (MAPE)	9.933617

Time series SES and DES of other electrical equipment





Residuals using SES and DES of other electrical equipment